ANSWERS

PHYSICS

1. (b)	2. (d)	3. (b)	4. (a)	5. (a)	6. (a)	7. (b)	8. (b)	9. (b)	10. (a)
11. (d)	12. (a)	13. (b)	14. (b)	15. (d)	16. (d)	17. (d)	18. (b)	19. (c)	20. (c)
21. (a)	22. (d)	23. (c)	24. (c)	25. (c)	26. (b)	27. (b)	28. (a)	29. (d)	30. (d)
31. (b)	32. (b)	33. (b)	34. (a)	35. (c)	36. (b)	37. (d)	38. (d)	39. (d)	40. (c)
41. (a)	42. (a)	43. (a)	44. (a)	45. (b)	46. (b)	47. (b)	48. (a)	49. (b)	50. (b)
51. (a)	52. (c)	53. (c)	54. (a)	55. (c)	56. (a)	57. (d)	58. (a)	59. (b)	60. (d)

CHEMISTRY

1. (c)	2. (c)	3. (d)	4. (c)	5. (d)	6. (a)	7. (c)	8. (b)	9. (a)	10. (c)
11. (b)	12. (b)	13. (d)	14. (b)	15. (b)	16. (d)	17. (c)	18. (b)	19. (a)	20. (c)
21. (c)	22. (b)	23. (a)	24. (b)	25. (c)	26. (a)	27. (b)	28. (b)	29. (b)	30. (d)
31. (d)	32. (b)	33. (c)	34. (a)	35. (d)	36. (c)	37. (b)	38. (d)	39. (b)	40. (a)
41. (c)	42. (c)	43. (d)	44. (a)	45. (d)	46. (a)	47. (b)	48. (a)	49. (d)	50. (a)
51. (d)	52. (c)	53. (a)	54. (c)	55. (c)	56. (b)	57. (b)	58. (a)	59. (d)	60. (b)

MATHEMATICS

1. (d)	2. (b)	3. (b)	4. (c)	5. (d)	6. (d)	7. (b)	8. (d)	9. (a)	10. (d)
11. (b)	12. (d)	13. (c)	14. (c)	15. (c)	16. (c)	17. (a)	18. (c)	19. (c)	20. (b)
21. (a)	22. (d)	23. (b)	24. (a)	25. (d)	26. (c)	27. (b)	28. (b)	29. (c)	30. (a)
31. (b)	32. (c)	33. (b)	34. (a)	35. (d)	36. (c)	37. (a)	38. (a)	39. (a)	40. (b)
41. (b)	42. (d)	43. (a)	44. (a)	45. (d)	46. (a)	47. (c)	48. (c)	49. (a)	50. (d)
51. (b)	52. (c)	53. (c)	54. (a)	55. (d)	56. (a)	57. (c)	58. (c)	59. (a)	60. (b)

1. All the components of electromagnetic spectrum have same velocity *ie*, 3×10^8 m/s.

2. According to photoelectric equation

$$E_k = h\nu - h\nu_0 \qquad ...(i)$$

If the energy of photon ($h\nu$) is less than the work function ($h\nu_0$) of metallic surface, then electrons will never be ejected from surface regardless of intensity of incident light.

Also, from Eq. (i), when

$$\nu = \nu_0, E_k = 0$$

∴ Graph (d) represents variation of E_k with ν.

3. Threshold energy of A is

$$E_A = h\nu_A$$
$$= 6.6 \times 10^{-34} \times 1.8 \times 10^{14}$$
$$= 11.88 \times 10^{-20} \text{ J}$$
$$= \frac{11.88 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV}$$
$$= 0.74 \text{ eV}$$

Similarly, $E_B = 0.91$ eV

Since, the incident photons have energy greater than E_A but less than E_B.

So, photoelectrons will be emitted from metal A only.

4. Ionization energy $= RchZ^2$

$Z = 3$ for Li^{2+}

∴ Ionization energy $= (3)^2\, Rch$

$= 9\, Rch$

5. The balanced condition for Wheatstones bridge is $\frac{P}{Q} = \frac{R}{S}$

as is obvious from the given values.

No, current flows through galvanometer is zero.

Now, P and R are in series, so

Resistance $R_1 = P + R$

$= 10 + 15 = 25\,\Omega$

Similarly, Q and S are in series, so

Resistance $R_2 = R + S$

$= 20 + 30 = 50\,\Omega$

Net resistance of the network as R_1 and R_2 are in parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\therefore \quad R = \frac{25 \times 50}{25 + 50} = \frac{50}{3}\,\Omega$$

Hence, $$I = \frac{V}{R} = \frac{6}{\frac{50}{3}} = 0.36 \text{ A}$$

6. Number of emitted spectral lines

$$N = \frac{n(n-1)}{2}$$

1st case

$$N = 3$$

$$\therefore \quad 3 = \frac{n_1(n_1 - 1)}{2}$$

$$\Rightarrow \quad n_1^2 - n_1 - 6 = 0$$

$$\Rightarrow (n_1 - 3)(n_1 + 2) = 0$$

$$n_1 = 3, n_1 = -2$$

Negative value of n_1 is not possible

$$\therefore \quad n_1 = 3$$

2nd case

$$N = 6$$

Again, $$6 = \frac{n_2(n_2 - 1)}{2}$$

$$\Rightarrow \quad n_2^2 - n_2 - 12 = 0$$

$$\Rightarrow (n_2 - 4)(n_2 + 3) = 0$$

$$n_2 = 4, n_2 = -3$$

Again, as n_2 is always positive

$$\therefore \quad n_2 = 4$$

Velocity of electron $v = \frac{Z e^2}{2 \varepsilon_0 hn}$

$$\therefore \quad \frac{\nu_1}{\nu_2} = \frac{n_2}{n_1}$$

$$\Rightarrow \quad \frac{\nu_1}{\nu_2} = \frac{4}{3}$$

7. The magnetic field produced at the centre of the circular coil carrying current is given by

$$B = \frac{\mu_0 NI}{2r}$$

For one turn $N = 1$

$$B_0 = \frac{\mu_0 I}{2r}$$

As the coil is rewound

$$r' = \frac{r}{3}, N' = 3$$

$$\therefore \quad B' = \frac{\mu_0 I \times 3}{2 \times \left(\frac{r}{3}\right)}$$

$$= \frac{9\mu_0 I}{2r} = 9B_0$$

8. For a moving charge in a perpendicular magnetic field,

$$\frac{mv^2}{r} = Bqv$$

$$\Rightarrow \quad r = \frac{mv}{Bq} = \frac{p}{Bq}$$

or $$\frac{r_p}{r_d} = \frac{p_p}{p_d} \quad \ldots(i)$$

(as q is same for both)

Also, momentum $p = \sqrt{2mE}$

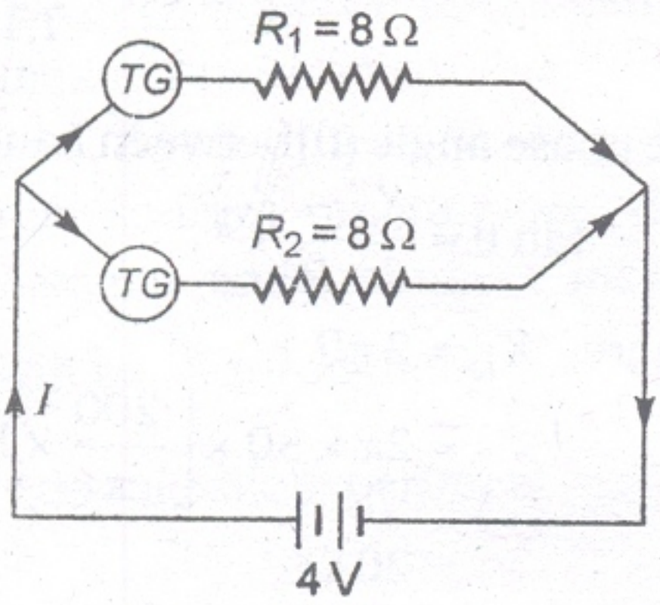
or $$\frac{p_p}{p_d} = \sqrt{\frac{m_p}{m_d}} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have,

$$\frac{r_p}{r_d} = \sqrt{\frac{m_p}{m_d}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

9. Current in tangent galvanometer

$$I = \frac{2rH}{\mu_0 N} \tan\theta \quad \ldots(i)$$



Here, R_1 and R_2 are in parallel

$$\therefore \quad \frac{1}{R_{net}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{R_2 + R_1}{R_1 R_2} = \frac{8+8}{8\times 8}$$

$$R_{net} = 4\,\Omega$$

Hence, $$I = \frac{V}{R} = \frac{4}{4} = 1\text{ A}$$

From Eq. (i), we get

$$\frac{r\tan\theta}{N} = \frac{\mu_0 I}{2H}$$

$$\therefore \quad \frac{r_A \tan\theta_A}{N_A} = \frac{r_B \tan\theta_B}{N_B}$$

$$\Rightarrow \quad \frac{8\times 1}{\sqrt{3}\times 2} = \frac{16\times\sqrt{3}}{N_B}$$

$$\therefore \quad N_B = 12 \text{ turns}$$

10. Lorentz force = centripetal force

ie, $$Bq\,v = \frac{mv^2}{r}$$

$$\Rightarrow \quad Bq = m\omega$$

$$\Rightarrow \quad Bq = m2\pi f \qquad (\text{as } v = r\omega)$$

$$\therefore \quad f = \frac{Bq}{2\pi m}$$

11. Loss of energy $= \frac{1}{2}\frac{C_1 C_2}{(C_1 + C_2)}(V_1 - V_2)^2$

$$= \frac{1}{2}\frac{5\times 10^{-6}\times 5\times 10^{-6}(2000-1000)^2}{(5+5)\times 10^{-6}}$$

$$= \frac{5\times 5}{2\times 10} = 1.25\text{ J}$$

12. The condenser with air as the dielectric has capacitance

$$C_1 = \frac{\varepsilon_0}{d}\left(\frac{3A}{4}\right) = \frac{3\varepsilon_0 A}{4d}$$

Similarly, the condenser with K as the dielectric constant has capacitance

$$C_2 = \frac{\varepsilon_0 K}{d}\left(\frac{A}{4}\right) = \frac{\varepsilon_0 AK}{4d}$$

Since, C_1 and C_2 are in parallel

$$C_{net} = C_1 + C_2$$

$$= \frac{3\varepsilon_0 A}{4d} + \frac{\varepsilon_0 AK}{4d}$$

$$= \frac{\varepsilon_0 A}{d}\left[\frac{3}{4} + \frac{K}{4}\right]$$

$$= \frac{C}{4}(K+3)$$

13. Drift velocity, $v_d = \frac{I}{neA}$

$$\Rightarrow v_d = \frac{5}{(5\times 10^{26})\times(1.6\times 10^{-19})\times(4\times 10^{-6})}$$

$$= \frac{1}{64} = 1.56\times 10^{-2}\text{ ms}^{-1}$$

14. Resistance of a bulb

$$R = \frac{V^2}{P}$$

$$\therefore \quad R_1 = \frac{(220)^2}{25} = 1936\,\Omega$$

$$R_2 = \frac{(220)^2}{100} = 484\,\Omega$$

Since, R_1 and R_2 are in series

$$R_{net} = R_1 + R_2$$
$$= 1936 + 484 = 2420\,\Omega$$

Hence, current $I = \frac{V}{R}$

$$= \frac{440}{2420} = \frac{2}{11}\,A$$

Potential difference across 25 W bulb

$$V_1 = IR_1$$
$$= \frac{2}{11} \times 1936$$
$$= 352\,V$$

Potential difference across 100 W bulb

$$V_2 = IR_2$$
$$= \frac{2}{11} \times 484$$
$$= 88\,V$$

Potential difference across 25 W bulb in this combination is 352 V, but it can tolerate only 220 V.

Hence, it will fuse.

15. Here, 2Ω and 2Ω are in parallel

$$\therefore \quad \frac{1}{R} = \frac{1}{2} + \frac{1}{2}$$

$$R = \frac{2 \times 2}{2 + 2} = 1\,\Omega$$

Now, internal resistance (1Ω), 2Ω, 4Ω and resistance R are in series.

$$\therefore \quad R_{net} = 1\,\Omega + 2\,\Omega + 4\,\Omega + 1\,\Omega$$
$$= 8\,\Omega$$

Hence, current $I = \frac{V}{R} = \frac{4}{8} = 0.5\,A$

16. The force between the conductors is attractive if the currents in them are in the same direction and repulsive if the currents are in opposite direction.

Mutual force between conductors A and C

$$F_1 = \frac{\mu_0}{2\pi}\frac{I_1 I_2 l}{r}$$
$$= \frac{\mu_0}{2\pi}\frac{2 \times 3 \times 1}{0.05}$$
$$= 2.40 \times 10^{-5}\,N \text{ (towards } A)$$

Mutual force between conductors B and C

$$= \frac{\mu_0}{2\pi} \times \frac{4 \times 3 \times 1}{0.08}$$
$$= 3 \times 10^{-5}\,N \text{ (towards } B)$$

Hence, the resultant force experienced by C

$$= (3 - 2.4) \times 10^{-5}\,N$$
$$= 0.6 \times 10^{-5}\,N \text{ (towards } B)$$

17. Resistance of bulb

$$R = \frac{V^2}{P} = \frac{(100)^2}{50}$$
$$= 200\,\Omega$$

Current through bulb $(I) = \frac{V}{R}$

$$= \frac{100}{200} = 0.5\,A$$

In a circuit containing inductive reactance (X_L) and resistance (R), impedance (Z) of the circuit is

$$Z = \sqrt{R^2 + \omega^2 L^2} \qquad ...(i)$$

Here, $Z = \frac{200}{0.5} = 400\,\Omega$

Now $X_L^2 = Z^2 - R^2$

$$= (400)^2 - (200)^2$$

$$(2\pi fL)^2 = 12 \times 10^4$$

$$L = \frac{2\sqrt{3} \times 100}{2\pi \times 50}$$

$$= \frac{2\sqrt{3}}{\pi} = 1.1\,H$$

18. The phase angle (θ) between I and V is given by

$$\tan\theta = \frac{X_L - X_C}{R} \qquad ...(i)$$

where, $X_L = 2\pi fL$

$$= 2\pi \times 50 \times \left[\frac{200}{\pi} \times 10^{-3}\right]$$

$$= 20\,\Omega$$

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1 \times \pi}{2\pi \times 50 \times 10^{-3}}$$

$$= 10\,\Omega$$

and $R = 10\,\Omega$

Substituting values of X_L, X_C and R in Eq. (i), we get

$$\tan\theta = \frac{20 - 10}{10} = 1$$

$$\Rightarrow \quad \tan\theta = \tan\frac{\pi}{4}$$

$$\therefore \quad \theta = \frac{\pi}{4}$$

The phase angle of the circuit is $\frac{\pi}{4}$.

19. Efficiency of transformer,

$$\eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\Rightarrow \quad \frac{88}{100} = \frac{880}{P_i}$$

$$\Rightarrow \quad P_i = 1000 \text{ W}$$

Input current, $I_P = \frac{P_i}{V_i}$

$$= \frac{1000}{2200}$$

$$= 0.45 \text{ A}$$

20. Induced emf, $e = \frac{-L\,dI}{dt}$

$$\therefore \quad 100 = \frac{-L[0-4]}{0.1}$$

$$\Rightarrow \quad L = \frac{100 \times 0.1}{4}$$

$$= 2.5 \text{ H}$$

21. The electromagnetic theory of light failed to explain photoelectric effect.

22. The rays of light from two coherent sources superimpose each other on the screen forming alternate maxima (with maximum intensity I_0) and minima (with intensity zero). If two non-coherent sources superimpose, there will be no maxima and minima, instead the intensity will be $\frac{I_0}{2}$ throughout.

23. In Young's double slit experiment

half angular width is given by

$$\sin\theta = \frac{\lambda}{d}$$

$$= \frac{589 \times 10^{-9}}{0.589 \times 10^{-3}} = 10^{-3}$$

$$\Rightarrow \quad \theta = \sin^{-1}(0.001)$$

24. $x = \frac{(2n+1)\lambda D}{2a}$

For red light $x = \frac{(4+1)D}{2a} \times 6500$

For unknown wavelength of light,

$$x = \frac{(6+1)D}{2a} \times \lambda$$

Accordingly

$$\therefore \quad 5 \times 6500 = 7 \times \lambda$$

$$\Rightarrow \quad \lambda = \frac{5}{7} \times 6500$$

$$= 4642.8 \text{ Å}$$

25. Distance of jeep, $x = \frac{D \times d}{1.22 \times \lambda}$

where D = diameter of lens,

d = seperation between sources.

$$\Rightarrow \quad x = \frac{(2 \times 10^{-3}) \times 1.2}{1.22 \times 5896 \times 10^{-10}}$$

$$= 3336 \text{ m}$$

$$\Rightarrow \quad x = 3.34 \text{ km}$$

26. According to de-Broglie hypothesis

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

$$\therefore \quad \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times (1.6 \times 10^{-27})(1.6 \times 10^{-19}) \times 1000}}$$

$$= \frac{6.6 \times 10^{-34}}{7.16 \times 10^{-22}}$$

$$= 0.9 \times 10^{-12} \text{ m}$$

27. To form its own isotope atomic number (Z) should remain same.

So, the emission of one α-particle and two β-particles will maintain the Z same.

where α-particle = ${}_2He^4$; β-particle = ${}_{-1}\beta^0$

28. We know, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

$$\Rightarrow \quad \frac{N}{10000} = \left(\frac{1}{2}\right)^{10/20}$$

$$\Rightarrow \quad N = \frac{10000}{\sqrt{2}} = \frac{10000}{1.414} = 7070$$

29. In Raman effect, Stokes' lines are spectral lines having lower frequency or greater wavelength than that of the original line.

30. Laser action involves all the given phenomenon
(i) Amplification of particular frequency
(ii) Population inversion
(iii) Stimulated emission

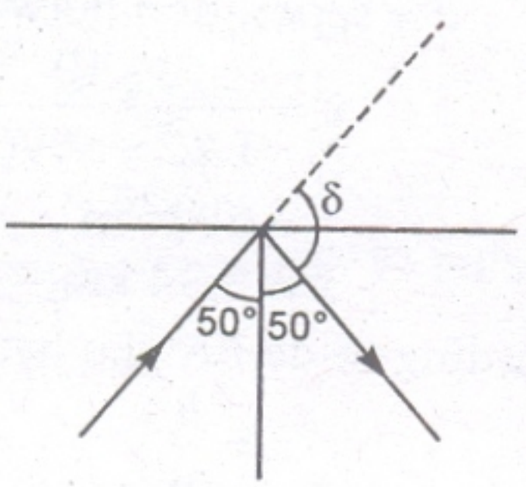
31. Refractive index, ${}^{a}\mu_{g} = 1.5$

$$\frac{1}{\sin C} = 1.5$$

$$\Rightarrow \quad C = 42^\circ$$

Critical angle for glass = 42°

When the angle of incidence in the denser medium is greater than the critical angle, reflection takes place inside the denser medium.



Hence, a ray of light incident at 50° in glass medium undergoes total internal reflection.

Deviation (δ) = 180° – (50° + 50°)
(from the figure)

or $\delta = 80^\circ$

32. Refractive index $\mu = \dfrac{\text{Real depth } (d)}{\text{Apparent depth } (x)}$

For 1st liquid, $\sqrt{2} = \dfrac{d}{x_1}$

$$\Rightarrow \quad x_1 = \frac{d}{\sqrt{2}}$$

Similarly, for 2nd liquid,

$$n = \frac{d}{x_2}$$

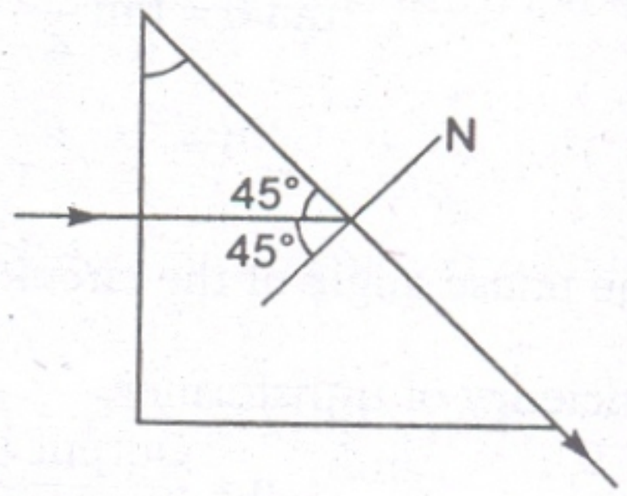
$$\Rightarrow \quad x_2 = \frac{d}{n}$$

Total apparent depth $= x_1 + x_2$

$$= \frac{d}{\sqrt{2}} + \frac{d}{n}$$

$$= \frac{d\,(n + \sqrt{2})}{n\sqrt{2}}$$

33. Refractive index, $\mu = \dfrac{1}{\sin C}$,
where C is the critical angle.



Here, $C = 45^\circ$ (as shown)

$\therefore \quad \mu = \dfrac{1}{\sin 45^\circ}$

or $\mu = \sqrt{2}$

or $\mu = 1.414$

34. Equivalent focal length (F) of two lenses separated by distance d is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{0.2} + \frac{1}{0.2} - \frac{0.5}{(0.2)\,(0.2)}$$

$$= 5 + 5 - 0.5 \times 5 \times 5$$

$$= 10 - 12.5$$

$$= -2.5$$

$$\therefore \quad F = -\frac{1}{2.5} = -0.4\ \text{m}$$

35. Dispersive power, $\omega = \dfrac{\delta_B - \delta_R}{\delta}$

For 1st prism, $\omega_1 = \dfrac{\delta_B - \delta_R}{\delta} = \dfrac{12^\circ - 8^\circ}{10^\circ} = \dfrac{2}{5}$

where $\delta = \dfrac{\delta_B + \delta_R}{2}$

Similarly, for 2nd prism,

$$\omega_2 = \frac{14^\circ - 10^\circ}{12} = \frac{1}{3}$$

$$\therefore \quad \frac{\omega_1}{\omega_2} = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$$

36. Refractive index of glass, $\mu_g = \tan\theta_P$
where θ_P = polarising angle

$$\Rightarrow \quad \mu_g = \tan 60^\circ = \sqrt{3}$$

Now, $\mu_g = \dfrac{c}{v_g}$

$$\therefore \quad \frac{c}{v_g} = \sqrt{3}$$

$$\Rightarrow \quad v_g = \frac{3 \times 10^8}{\sqrt{3}}$$

$$= \sqrt{3} \times 10^8\ \text{m/s}$$

37. For liquid A

$L_1 = 20$ cm, $\theta_1 = 38°$; concentration $= C_1$

Specific rotation $\alpha_1 = \dfrac{\theta_1}{L_1 C_1}$

$$= \frac{38°}{20 \times C_1}$$

Similarly, for liquid B

$L_2 = 30$ cm, $\theta_2 = -24°$, concentration $= C_2$

Specific rotation $\alpha_2 = \dfrac{\theta_2}{L_2 C_2}$

$$= \frac{(-24°)}{30 \times C_2}$$

The mixture has 1 part of liquid A and 2 parts of liquid B.

$\therefore \quad C_1' : C_2' = 1 : 2$

$$\theta = [\alpha_1 C_1' + \alpha_2 C_2'] l$$

$$= \left\{ \frac{38°}{20 \times C_1} \times \frac{C_1}{3} + \frac{(-24°)}{30 \times C_2} \times \frac{2C_2}{3} \right\} \times 30$$

$$= 19° - 16° = 3°$$

Thus, the optical rotation of mixture is + 3° in right hand direction.

38. Coulomb force is given by

$$F = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2}$$

$$\therefore \quad (10 \times 10^{-3}) \times 10 = \frac{(9 \times 10^9) \times q^2}{(0.6)^2}$$

$[\because q_1 = q_2 = q]$

or $$q^2 = \frac{10^{-1} \times 0.36}{9 \times 10^9}$$

$$= 4 \times 10^{-12}$$

$\therefore \quad q = 2 \times 10^{-6}\text{C} = 2\mu\text{C}$

39. Potential $(V) = 3x^2 + 5$

Intensity of the electric field $= \dfrac{dV}{dx} = 6x$

$\therefore \quad E$ at $x = -2$

$$= 6(-2) = -12\ \text{V/m}$$

40. Volume of 8 drops = volume of big drop

$\therefore \quad \left(\dfrac{4}{3}\pi r^3\right) \times 8 = \dfrac{4}{3}\pi R^3$

$\Rightarrow \quad 2r = R \qquad \ldots(i)$

According to charge conservation

$$8q = Q \qquad \ldots(ii)$$

Potential of one small drop $(V') = \dfrac{q}{4\pi \varepsilon_0 r}$

Similarly, potential of big drop $(V) = \dfrac{Q}{4\pi \varepsilon_0 R}$

Now, $$\frac{V'}{V} = \frac{q}{Q} \times \frac{R}{r}$$

$\Rightarrow \quad \dfrac{V'}{20} = \dfrac{q}{8q} \times \dfrac{2r}{r}$

[from Eqs. (i) and (ii)]

$\therefore \quad V' = 5\,\text{volt}$

41. Impulse = force × time

$$= [\text{MLT}^{-2}][\text{T}]$$

$$= [\text{MLT}^{-1}]$$

42. Maximum height, $H_0 = \dfrac{u^2 \sin^2\theta}{2g}$

Range, $R = \dfrac{u^2 \sin 2\theta}{g}$

Given, $H_0 = \dfrac{R}{2}$

$\therefore \quad \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{u^2 2\sin\theta\cos\theta}{2g}$

$\Rightarrow \quad \sin\theta = 2\cos\theta$

$\Rightarrow \quad \tan\theta = 2$

$\therefore \quad \theta = \tan^{-1}(2)$

43. Total mass of the shell = 20 kg

Ratio of the masses of the fragments = 2 : 3

$\therefore$ Masses of the fragments are 8 kg and 12 kg

Now, according to the conservation of momentum

$$m_1 v_1 = m_2 v_2$$

$\therefore \quad 8 \times 6 = 12 \times v$

v (velocity of the larger fragment) = 4 m/s

Kinetic energy $= \dfrac{1}{2} m v^2$

$$= \frac{1}{2} \times 12 \times (4)^2 = 96\,\text{J}$$

44. The rise or fall of liquids in vertical capillary tubes is called capillarity. Water in plant fibres rises due to same phenomenon.

45. The acceleration due to gravity

$$g = \frac{GM}{R^2}$$

At a height h above the earth's surface, the acceleration due to gravity is

$$g' = \frac{GM}{(R+h)^2}$$

$$\therefore \quad \frac{g}{g'} = \left(\frac{R+h}{R}\right)^2 = \left(1+\frac{h}{R}\right)^2$$

$$\frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

$$= \left(1-\frac{2h}{R}\right)$$

but $g' = \frac{g}{2}$ (given)

$$\therefore \quad \frac{g/2}{g} = 1-\frac{2h}{R}$$

$$\frac{2h}{R} = \frac{1}{2}$$

$$h = \frac{R}{4}$$

46. According to equation of continuity,

$$av = \text{constant}$$

$\therefore$ For tube, $(8\times10^{-4})\times\left(\frac{0.15}{60}\right) = a_1v_1$

For holes, $(40\times10^{-8})\times v = a_2v_2$

$$\therefore \quad a_2v_2 = a_1v_1$$

$$\therefore \quad 40\times10^{-8}\times v = \frac{8\times10^{-4}\times0.15}{60}$$

$$\Rightarrow \quad v = \frac{8\times10^{-4}\times0.15}{40\times10^{-8}\times60}$$

$$= 5\ \text{m/s}$$

47. Equation of an adiabatic process is

$$pV^{\gamma} = \text{constant} \qquad \ldots(i)$$

Given, $p^3 = \frac{k}{V^4}$

$$P^3V^4 = k \quad \text{(constant)}$$

$$\Rightarrow \quad pV^{4/3} = k \qquad \ldots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$\gamma = \frac{4}{3} = 1.33$$

48. Let the temperature of the junction be θ.

Heat flow through the rod is given by

$$Q = \frac{KA(\theta_1-\theta_2)t}{d}$$

Here, $K_1(100-\theta) = K_2(\theta-25)$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{\theta-25}{100-\theta}$$

But $\frac{K_1}{K_2} = \frac{2}{3}$ (given)

$$\therefore \quad \frac{2}{3} = \frac{\theta-25}{100-\theta}$$

$$\Rightarrow \quad 3\theta-75 = 200-2\theta$$

$$\Rightarrow \quad 5\theta = 275$$

$$\theta = 55^\circ\text{C}$$

49. At constant pressure,

Heat required $= nC_p\Delta T$

$$\Rightarrow \quad 310 = 2\times C_p\times(35-25)$$

$$\Rightarrow \quad C_p = \frac{310}{20} = 15.5\ \text{J/mol/K}$$

Similarly, at constant volume,

Heat required $= nC_V\Delta T$

$= 2(C_p-R)\times(35-25)$ $[\because C_p-C_V=R]$

$= 2\times(15.5-8.3)\times10$

$= 2\times7.2\times10$

$= 144\ \text{J}$

50. Efficiency $\eta = 1-\frac{T_2}{T_1}$

$$= 1-\frac{(273+27)}{(273+127)}$$

$$= 1-\frac{300}{400}$$

$$= \frac{1}{4}$$

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$\frac{1}{4} = \frac{W}{40}$$

$$\Rightarrow \quad W = 10\ \text{kJ}$$

51. For a wave,

$$y = a\sin\frac{2\pi}{\lambda}(vt-x) \qquad \ldots(i)$$

Differentiating Eq (i) w.r.t. t, we get

$$\frac{dy}{dt} = \frac{2\pi va}{\lambda}\cos\frac{2\pi}{\lambda}(vt-x)$$

Now, maximum velocity is obtained when

$$\cos\frac{2\pi}{\lambda}(vt-x) = 1$$

$$\therefore \quad v_{\max} = \left(\frac{dy}{dt}\right)_{\max} = \frac{2\pi va}{\lambda}$$

but $v_{\max} = \frac{v}{2}$ (given)

$$\therefore \quad \frac{\nu}{2} = \frac{2\pi\nu a}{\lambda}$$

$$\Rightarrow \quad a = \frac{\lambda}{4\pi}$$

52. Velocity of sound in a gas

$$\nu = \sqrt{\frac{\gamma p}{d}}$$

$$\therefore \quad \frac{\nu_{H_2}}{V_{He}} = \sqrt{\frac{\gamma_{H_2} \times d_{He}}{d_{H_2} \times \gamma_{He}}}$$

$$\frac{H_2}{He} = \sqrt{\frac{7 \times 3 \times 2}{5 \times 5}}$$

As $\quad \frac{d_{He}}{d_{H_2}} = 2$

$$\therefore \quad \frac{\nu_{H_2}}{\nu_{He}} = \frac{\sqrt{42}}{5}$$

53. The reflected sound appears to propagate in a direction opposite to that of moving engine. Thus, the source and the observer can be presumed to approach each other with same velocity.

$$\nu' = \frac{\nu(\nu + \nu_o)}{(\nu - \nu_s)}$$

$$= \nu\left(\frac{\nu + \nu_s}{\nu - \nu_s}\right) \qquad [\because \nu_o = \nu_s]$$

$$\Rightarrow \quad \nu' = 1.2\left(\frac{350 + 50}{350 - 50}\right)$$

$$= \frac{1.2 \times 400}{300}$$

$$= 1.6 \text{ kHz}$$

54. For fundamental frequency of vibration of air column

$$l = \frac{\lambda}{2}$$

$$\Rightarrow \quad \lambda = 2l$$

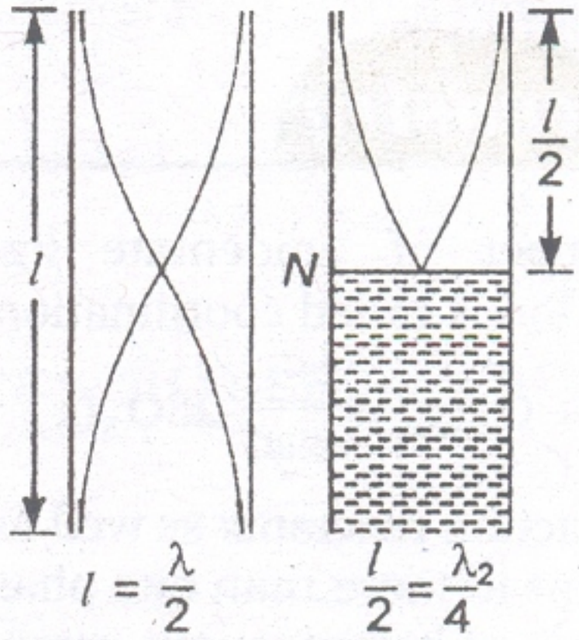
Frequency, $\quad f = \frac{\nu}{\lambda}$

$$\Rightarrow \quad f = \frac{\nu}{2l} \qquad \ldots(i)$$

When tube is placed inside water with half length immersed in water then

$$\frac{l}{2} = \frac{\lambda'}{4}$$

$$\Rightarrow \quad \lambda' = 2l$$



Frequency $\quad f' = \frac{\nu}{\lambda'} = \frac{\nu}{2l} \qquad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$f' = f$$

55. By using Wien's displacement law,

$$(\lambda_m)_1 T_1 = (\lambda_m)_2 T_2$$

$$\Rightarrow \quad 500 \times 6000 = 400 \times T_2$$

$$\Rightarrow \quad T_2 = \frac{500 \times 6000}{400}$$

$$T_2 = 7500 \text{ K}$$

56. Radius of nucleus $R = R_0 A^{1/3}$

where $R_0 = 1.2 \times 10^{-15}$ m

Volume of nucleus $(V) = \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi [R_0 A^{1/3}]^3$$

$$= \frac{4}{3}\pi R_0^3 A$$

$$\therefore \quad V \propto A$$

57. Leptons is a class of elementary particles. They interact by the electromagnetic interaction and the weak interaction.

An electron is a lepton.

58. In *p*-type semiconductor density of mobile holes exceeds that of conduction electrons. Hence, minority carriers in *p*-type semiconductor are conduction (free) electrons.

59. Reverse resistance

$$= \frac{\Delta V}{\Delta I} = \frac{1}{0.5 \times 10^{-6}}$$

$$= 2 \times 10^6 \Omega$$

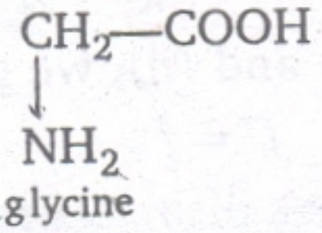
60. The output Y is a combination of AND + NOT gate. Hence, the truth table is for NAND gate.

1. The number of unidentate ligand in the complex ion is called coordination number.

2. $2SO_2(g) + O_2(g) \underset{\text{(solid)}}{\overset{V_2O_5}{\rightleftharpoons}} 2SO_3(g)$

   In this reaction reactants as well as the catalyst are present in more than one phase hence it is an example of heterogenous catalysis.

3. Glycine is optically inactive amino acid due to absence of chiral carbon atom.



   glycine

4. Bond order $\propto$ Stability

   Hence, for a stable molecule the value of bond order must be positive. When bond order is zero the molecule will not exist.

5. The reaction is said to be of second order if its reaction rate is determined by the variation of two concentration terms of reactants.

   $CH_3COOCH_3 + NaOH \longrightarrow CH_3COONa + H_2O$

   is an example of second order reaction.

6. Ethyl alcohol is made unfit for drinking purpose by adding some poisonous substance as methanol, naphtha, pyridine, rubber etc. It is called denatured alcohol.

7. During the formation of a chemical bond energy increases

8. $$KE = \frac{3}{2}RT$$

   $$KE \propto T$$

   $$\frac{KE_{O_2}}{KE_{SO_2}} = \frac{T_{O_2}}{T_{SO_2}} = \frac{273}{546} = \frac{1}{2}$$

   $$KE_{SO_2} = 2\,KE_{O_2}$$

   $\therefore$ $$KE_{SO_2} > 2\,KE_{O_2}$$

9. $+I$ effect is shown by $-CH_3$ while $-I$ effect is shown by —Br, —Cl, and $-NO_2$.

10. Formation of coloured solution is possible when metal ion in the compound contains unpaired electrons. *eg,*

    $Cu^+ : 3d^{10}\,4s^0$ colourless

    $Cu^{2+} : 3d^9\,4s^0$ blue

11. Benzene reacts with chlorine in presence of sunlight to give gammexane or benzene hexa chloride.

    $$C_6H_6 + 3Cl_2 \xrightarrow{\text{Sunlight}} C_6H_6Cl_6$$

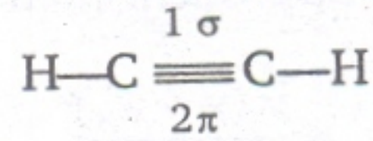
12. In the periodic table metals usually used as catalysts belong to *d*-block *eg,* Ni, Pt etc.

13. When a mixture of two or more non-reacting gases are enclosed in a container then the total pressure exerted by the gaseous mixture is equal to the sum of partial pressure of the components in the mixture.

    eg, $CO + H_2$ are non-reacting gases. Hence, Dalton's law of partial pressure is applicable to this system.

14. The general formula of a cycloalkane is C_nH_{2n}.

15. The structure of acetylene is



    Hence, in acetylene molecule, there are one sigma and two pi bonds are present between carbon atoms.

16. The properties of the system whose value is independent of the amount of substance present in the system are called intensive properties. *eg,* viscosity, surface tension, temperature, pressure etc.

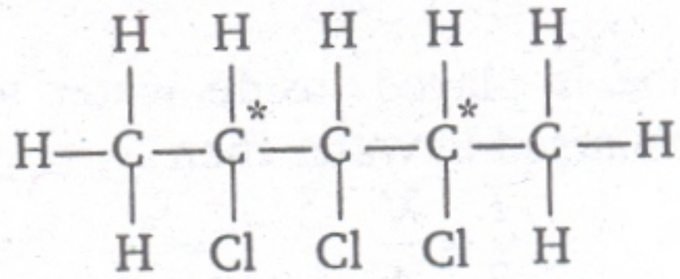
17. Hofmann's bromamide reaction is used to convert amide to amine.

    $$\underset{\text{amide}}{RCONH_2} + Br_2 + 4KOH \longrightarrow \underset{\text{amine}}{RNH_2} + K_2CO_3 + 2KBr + 2H_2O$$

18. The IUPAC name of $Na_3[Co(NO_2)_6]$ is sodium hexanitrocobaltate (III).

19. In the equilibrium state the value of ΔG is zero.

20. The structure of 2, 3, 4-trichloro pentane is



    Hence, two chiral carbon atoms are present in 2, 3, 4-trichloropentane.

21. Ethyl alcohol shows functional isomerism with dimethyl ether.

$\underset{\text{alcohol}}{C_2H_5OH}$ $\underset{\text{ether}}{CH_3—O—CH_3}$

22. $BiO_3^- + 6H^+ + 2e^- \longrightarrow Bi^{3+} + 3H_2O$

23. Orthophosphoric acid (H_3PO_4) is a tribasic acid.

$\because$ Normality = Molarity × basicity

$\therefore$ Normality = 3M × 3

= 9 N

24. Acidified sodium fusion extract on addition of ferric chloride solution gives blood red colouration, which confirms the presence of N and S.

$$3NaCNS + aq \cdot FeCl_3 \longrightarrow \underset{\substack{\text{red}\\(\text{ferric thiocyanide})}}{Fe(CNS)_3} + 3NaCl$$

25. $m = 10 \text{ mg} = 10 \times 10^{-6} \text{ kg}$

$v = 100 \text{ ms}^{-1}$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{10 \times 10^{-6} \times 100}$$

$= 6.63 \times 10^{-31}$ m

26. Angle strain in cyclopropane is 24°44′

$$\theta = \left[180 - \frac{360}{n}\right]$$

$$= \left[180 - \frac{360}{3}\right] = 180 - 120 = 60°$$

$$\text{Angle strain } \alpha = \frac{1}{2}[109°28° - \theta]$$

$$= \frac{1}{2}[109°28' - 60°]$$

$= 24°44'$

27. The electronic configuration of O_2^{2-} ion is

$O_2^{2-} — \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2, \sigma 2p_z^2, \pi 2p_x^2, \pi 2p_y^2, \overset{*}{\pi} 2p_x^2, \overset{*}{\pi} 2p_y^2$

Hence, number of antibonding electron pairs in O_2^{2-} molecular ion are four.

28. $[H^+][OH^-] = 10^{-14}$

$[10^{-2}][OH^-] = 10^{-14}$

$$[OH^-] = \frac{10^{-14}}{10^{-2}}$$

$= 10^{-12}$ mol dm^{-3}

29. Geometrical isomerism is shown by $>C=C<$ only when identical groups are not present on the double bonded carbon atoms.

30. Let the oxidation state of iron in $K_4[Fe(CN)_6]$ is x.

$4(+1) + x + 6(-1) = 0$

$4 + x - 6 = 0$

$x = +2$

31. $4Au + 8CN^- + 2H_2O + O_2 \longrightarrow \underset{\text{soluble}}{4[Au(CN)_2]^-} + 4OH^-$

$2[Au(CN)_2]^- + Zn \longrightarrow 2Au(s)\downarrow + [Zn(CN)_4]^{2-}$

32. Number of gram molecules $= \frac{6.02 \times 10^{25}}{6.02 \times 10^{23}}$

$= 100$

33. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds.

34. Paracetamol is an antipyretic. It is used to lower the temperature of the body in high fever.

35. Number of oxygen atom in 2 g of CO

$$= \frac{2}{28} \times 6.022 \times 10^{23} \times 1$$

Number of oxygen atom in 2 g of CO_2

$$= \frac{2}{44} \times 6.022 \times 10^{23} \times 2$$

Number of oxygen atom in 2 g of SO_2

$$= \frac{2}{64} \times 6.022 \times 10^{23} \times 2$$

Number of oxygen atom in 2 g of H_2O

$$= \frac{2}{18} \times 6.022 \times 10^{23} \times 1$$

Hence, 2 g of H_2O has maximum number of atoms of oxygen.

36. Isoelectronic species have same number of electron. Mg^{2+} and Na^+ both have 10 electrons hence, they are isoelectronic species.

37. Gram molecular volume of oxygen at STP is 5.6 L or 5600 cm^3.

38. Presence of halogen in organic compound can be detected by Beilstein's test.

39. Cr(24) : [Ar] $3d^5\ 4s^1$

Cr^{3+} : [Ar] $3d^3\ 4s^0$

40. $\underset{190}{SnCl_2} + \underset{71}{Cl_2} \longrightarrow SnCl_4$

$$\frac{190}{E_1} = \frac{71}{35.5}$$

$$E_1 = 95$$

41. Sc^{3+} has completely empty *d*-orbitals hence, it forms a colourless solutions in aqueous medium.

$Sc\,(21) : [Ar]\ 3d^1\ 4s^2$

$Sc^{3+} : [Ar]\ 3d^0\ 4s^0$

42. Hydrophobic sol are irreversible in nature. They have no affinity between the dispersed phase and the dispersion medium (H_2O). Further once precipitated, they do not form the colloidal sol by simple addition of water.

43. When an alkyl halide reacts with alcoholic ammonia in a sealed tube then a mixture of primary, secondary and tertiary amine is formed.

$$RX + NH_3 \longrightarrow \underset{pri\text{-amine}}{RNH_2} + HX$$

$$RNH_2 + XR \longrightarrow \underset{sec\text{-amine}}{R_2NH} + HX$$

$$R_2NH + XR \longrightarrow \underset{ter\text{-amine}}{R_3N} + HX$$

44. When conc. H_2SO_4 is heated with P_2O_5, the acid is converted into sulphur trioxide.

$$2H_2SO_4 + 2P_2O_5 \longrightarrow \underset{\text{sulphur trioxide}}{2SO_3} + 4HPO_3$$

45. According to second law of thermodynamics, the entropy of the universe always increases in the course of every spontaneous (natural) change.

46. KCN is a salt of weak acid and strong base hence, on being dissolved in water it gives basic solution *ie,* pH > 7 at 25°C.

47. In Clemmensen's reduction

Zn—Hg/ conc. HCl is used.

$$\rangle C{=}O + 4H \xrightarrow{Zn\text{-}Hg + \text{conc. HCl}} \rangle CH_2 + H_2O$$

This method is used to convert carbonyl compound into alkane.

48. When KBr is dissolved in water K^+ and Br^- are hydrated.

49. When the mixture of noble gas is cooled in a coconut bulb at 173 K then Ar, Kr and Xe are adsorbed on charcoal while He and Ne are not adsorbed.

50. Let *V* litre of 10 N HCl be mixed with (1 – *V*) litre of 4 N HCl to give (*V* + 1 – *V*) = 1 L of 7 N HCl.

$$N_1V_1 + N_2V_2 = N\,V$$

$$10\,V + 4\,(1 - V) = 7 \times 1$$

$$10\,V + 4 - 4\,V = 7$$

$$6\,V = 7 - 4$$

$$V = \frac{3}{6} = 0.50\ L$$

Volume of 10 N HCl = 0.50 L

Volume of 4 N HCl = 1 – 0.50 = 0.50 L

51. Vitamin B_{12} contains cobalt metal. The chemical name of vitamin B_{12} is cyanocobalamin.

52. Gram equivalent of element

= Gram equivalent of O_2

$$\frac{80}{E} = \frac{20}{8}$$

$$E = 32$$

53. $\underset{\text{methyl amine}}{CH_3NH_2} \xrightarrow[-HI]{CH_3I} \underset{\text{dimethyl amine}}{(CH_3)_2NH} \xrightarrow[-HI]{CH_3I}$

$\underset{\text{trimethyl amine}}{(CH_3)_3N}$

$\downarrow CH_3I$

$(CH_3)_4N^+I^-$

tetramethyl ammonium iodide

Hence, three molecules of CH_3I is used.

54. The relationship between ΔH and ΔU is

$$\Delta H = \Delta U + \Delta\, nRT$$

55. Limonite — $Fe_2O_3 \cdot 3H_2O$

Siderite — $FeCO_3$

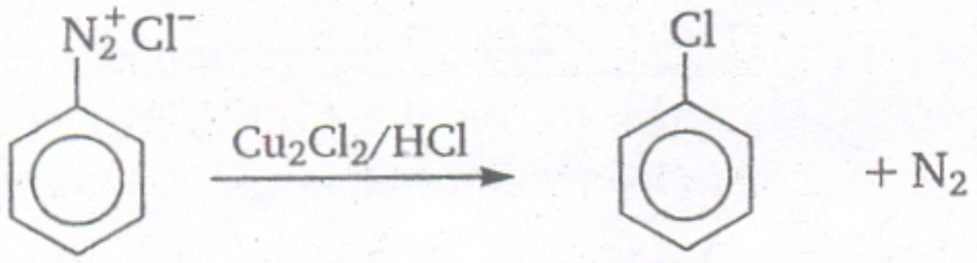
Carnallite — $KCl \cdot MgCl_2 \cdot 6H_2O$

Chalcopyrites — $CuFeS_2$

56. The order of entropy in solid, liquid and gas is

gas > liquid > solid

Hence, in sublimation of naphthalene, maximum increase in entropy is observed.

57. 

benzene diazonium chloride — chlorobenzene

This reaction is known as Sandmeyer's reaction.

58. $[CoCl_3(NH_3)_3]$ cannot ionise in solution because three chloride ions satisfy primary and secondary valencies. It will not be precipitated by the addition of $AgNO_3$.

59. ΔH = – ve (combustion reaction)
ΔS = – ve (spontaneous at all temperature)
ΔG = – ve (because reaction is spontaneous)

60. $>C=O + H_2NOH \longrightarrow >C=NOH + H_2O$
carbonyl compound — hydroxyl amine — oxime

Mathematics

1. We know that a vector perpendicular to $\vec{a}$ and in the plane containing $\vec{b}$ and $\vec{c}$ is given by $\vec{a} \times (\vec{b} \times \vec{c})$. Given $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} + 5\hat{j} - \hat{k}$

$$\therefore \quad \vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -5 \\ 3 & 5 & -1 \end{vmatrix} = 23\hat{i} - 14\hat{j} - \hat{k}$$

$$\text{Now, } \vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 23 & -14 & -1 \end{vmatrix}$$

$$= (-3 - 14)\hat{i} - \hat{j}(-2 + 23) + \hat{k}(-28 - 69)$$

$$= -17\hat{i} - 21\hat{j} - 97\hat{k}$$

Which is the required vector.

2. We have magnitudes of $\vec{OA}$ and $\vec{OB}$ are 5 and 6 respectively.
And $\angle BOA = 60°$

Now, $\vec{OA} \cdot \vec{OB} = |\vec{OA}| \cdot |\vec{OB}| \cdot \cos\theta$

$\Rightarrow \quad \vec{OA} \cdot \vec{OB} = 5 \cdot 6 \cdot \cos 60°$

$\Rightarrow \quad \vec{OA} \cdot \vec{OB} = 30 \times \frac{1}{2}$

$\Rightarrow \quad \vec{OA} \cdot \vec{OB} = 15$

3. We know that a vector perpendicular to the plane containing the points $\vec{A}$, $\vec{B}$, $\vec{C}$ is given by $\vec{A} \times \vec{B} + \vec{B} \times \vec{C} + \vec{C} \times \vec{A}$.

We have, $\vec{A} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{B} = 2\hat{i} + 0\hat{j} - \hat{k}$

and $\vec{C} = 0\hat{i} + 2\hat{j} + \hat{k}$

$$\text{Now, } \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \hat{i} + 5\hat{j} + 2\hat{k}$$

$$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\vec{C} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 5\hat{i} + \hat{j} - 2\hat{k}$$

Thus,

$$\vec{A} \times \vec{B} + \vec{B} \times \vec{C} + \vec{C} \times \vec{A} = (\hat{i} + 5\hat{j} + 2\hat{k}) + (2\hat{i} - 2\hat{j} + 4\hat{k}) + (5\hat{i} + \hat{j} - 2\hat{k})$$

$$= 8\hat{i} + 4\hat{j} + 4\hat{k}$$

4. Given, $\frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \ldots + \frac{1}{(3n-1)(3n+2)}$

$$= \frac{1}{3}\left[\left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{11}\right) + \ldots + \left(\frac{1}{3n-1} - \frac{1}{3n+2}\right)\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right]$$

$$= \frac{3n+2-2}{6(3n+2)} = \frac{n}{6n+4}$$

5. The general term of the expansion $(x + a)^n$ is $T_{r+1} = {}^nC_r x^{n-r} a^r$. We have $\left(3x - \frac{1}{2x}\right)^8$. Here,

$r = 8,\ x = 3x,\ a = \left(-\frac{1}{2x}\right),\ n = 8$

$\therefore$ Nineth term $T_9 = {}^8C_8 (3x)^{8-8} \left(\frac{-1}{2x}\right)^8$

$$= \frac{1}{256 \cdot x^8}$$

6. We have, $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

$$\Rightarrow \tan^{-1} x + 2\tan^{-1} \frac{1}{x} = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1}\left(\frac{2\left(\frac{1}{x}\right)}{1 - \left(\frac{1}{x}\right)^2}\right) = \frac{2\pi}{3}$$

$$\left[\because 2\tan^{-1} x = \tan^{-1} \frac{2x}{1 - x^2}\right]$$

$$\Rightarrow \tan^{-1} x + \tan^{-1}\left(\frac{2x}{x^2 - 1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{x + \frac{2x}{x^2 - 1}}{1 - \frac{2x^2}{x^2 - 1}}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \frac{x^3 - x + 2x}{x^2 - 1 - 2x^2} = \tan\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \frac{x^3 + x}{-1 - x^2} = \tan\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \frac{x(x^2 + 1)}{-1(x^2 + 1)} = -\sqrt{3}$$

$$\Rightarrow x = \sqrt{3}$$

7. We have, $\sin^2 17.5° + \sin^2 72.5°$

$= \sin^2 17.5° + \sin^2(90° - 17.5°)$

$= \sin^2 17.5° + \cos^2 17.5°$

$= 1 = 1^2$

$= \tan^2 45°$

8. Given complex number is $\frac{(1 + i)^2}{1 - i}$

$$= \frac{(1 + i^2 + 2i)}{1 - i} \times \frac{1 + i}{1 + i}$$

$$= \frac{2i(1 + i)}{1 - i^2}$$

$$= \frac{2i + 2i^2}{1 + 1}$$

$$= \frac{2i - 2}{2}$$

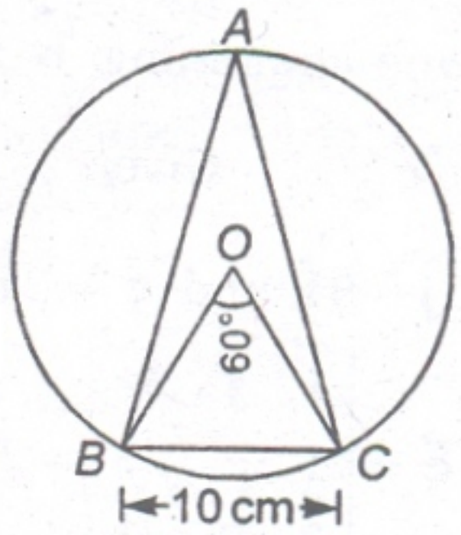
$$= i - 1$$

$\therefore$ Required conjugate is $-i - 1$

9. In ΔABC, $\angle A = 30°$

$BC = 10$ cm

O is the centre of circle



$\therefore$ $\angle BOC = 60°$

and OB and OC are the radius

$\therefore$ $\angle OBC = \angle OCB = 60°$

$\Rightarrow \Delta OBC$ is an equilateral triangle

$\therefore$ Radius of circle is $OB = OC = BC = 10$ cm

Now, area of the circumcircle is πr^2

$= \pi (10)^2 = 100\pi$ sq cm

10. We have, $\sin 3\theta = \sin \theta$

$$\Rightarrow \sin 3\theta - \sin \theta = 0$$

$$\Rightarrow 2\cos\left(\frac{3\theta + \theta}{2}\right) \sin\left(\frac{3\theta - \theta}{2}\right) = 0$$

$$\Rightarrow \cos 2\theta \cdot \sin \theta = 0$$

$$\Rightarrow \cos 2\theta = 0 \text{ or } \sin \theta = 0,\ \pi,\ 2\pi$$

$$\Rightarrow \cos 2\theta = \cos\left(\frac{\pi}{2}\right) \text{ or } \theta = 0,\ \pi,\ 2\pi$$

$$\Rightarrow 2\theta = \frac{\pi}{2} \text{ or } \theta = 0,\ \pi,\ 2\pi$$

$$\Rightarrow \theta = \frac{n\pi}{4} \text{ or } \theta = 0,\ \pi,\ 2\pi$$

$\because$ $-2\pi < \theta < 2\pi$

$\therefore$ $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ or $\theta = 0,\ \pi,\ 2\pi$

Thus, total number of solutions $= 7$

11. Since, the number of vertices of odd degree is always even and number of even degree is always even. So, $m + n$ is an even number.

12. Given, ellipse is $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Here, $a^2 = 36, b^2 = 16$

Since, $a > b$, so the sum of the focal distance of any point P on the ellipse is $PS + PS' = 2a$

$\Rightarrow PS + PS' = 2 \times 6$

$\Rightarrow PS + PS' = 12$

13. We have, $\sin\left[2\cos^{-1}\frac{\sqrt{5}}{3}\right]$

$= \sin\left[\cos^{-1}\left(2\cdot\left(\frac{\sqrt{5}}{3}\right)^2 - 1\right)\right]$

$[\because 2\cos^{-1} x = \cos^{-1}(2x^2 - 1)]$

$= \sin\left[\cos^{-1}\left(\frac{1}{9}\right)\right]$

$= \sin\left[\sin^{-1}\sqrt{1 - \left(\frac{1}{9}\right)^2}\right]$

$[\because \cos^{-1} x = \sin^{-1}(\sqrt{1 - x^2})]$

$= \sin\left[\sin^{-1}\sqrt{\frac{80}{81}}\right]$

$= \frac{\sqrt{80}}{9}$

$= \frac{4\sqrt{5}}{9}$

14. Given, $\frac{x^2}{36} - \frac{y^2}{k^2} = 1$

$\Rightarrow \frac{y^2}{k^2} = \frac{x^2}{36} - 1$

$\Rightarrow k^2 = \frac{36\,y^2}{x^2 - 36}$

$k^2 > 0$ if $x^2 - 36 > 0$

$\Rightarrow x^2 > 36$

This is true only for point (10, 4). So, (10, 4) lies on the hyperbola.

15. The given equation of parabola is $y = 2x^2 + x$

$\Rightarrow x^2 + \frac{x}{2} = \frac{y}{2}$

$\Rightarrow x^2 + \frac{x}{2} + \frac{1}{16} = \frac{y}{2} + \frac{1}{16}$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{2}\left(y + \frac{1}{8}\right)$

It can be rewritten as $X^2 = \frac{1}{2}Y$ ...(i)

where $x + \frac{1}{4} = X$ and $y + \frac{1}{8} = Y$

On comparing with $X^2 = 4AY$, we get $A = \frac{1}{8}$

focus of Eq.(i) is $\left(0, \frac{1}{8}\right)$ *ie*, $X = 0, Y = \frac{1}{8}$

$\Rightarrow x + \frac{1}{4} = 0, y + \frac{1}{8} = \frac{1}{8} \Rightarrow x = -\frac{1}{4}, y = 0$

$\therefore$ Focus of given parabola is $\left(-\frac{1}{4}, 0\right)$.

16. Given, $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and

$10\,B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

Since, B is the inverse of A

ie, $B = A^{-1}$

So, $10\,A^{-1} = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow 10\,A^{-1}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} A$

$\Rightarrow 10\,I = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ -5+\alpha & 5+\alpha & -5+\alpha \\ 0 & 0 & 10 \end{bmatrix}$

$\Rightarrow -5 + \alpha = 0$

$\Rightarrow \alpha = 5$

Alternative : $A^{-1} = \frac{1}{|A|}\cdot \text{adj}\,(A)$

$$\text{adj}\,(A) = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$|A| = 1[1 + 3] + 1[2 + 3] + 1[2 - 1] = 10$

$$\therefore \quad A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow \quad \alpha = 5$$

17. We have, $A = \begin{bmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{bmatrix}$

Since, A is singular

$$\therefore \quad |A| = 0$$

$$\Rightarrow \quad 0 - x[x^2 - 0] + 16[9x - 0] = 0$$

$$\Rightarrow \quad -x^3 + 144x = 0$$

$$\Rightarrow \quad x(x^2 - 144) = 0$$

$$\Rightarrow \quad x = 0, x = \pm 12$$

18. Given, $A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$C_{11} = 8 - 6 = 2$

$C_{12} = -(0 + 9) = -9, C_{31} = +6 - 4 = 2$

$C_{13} = 0 - 6 = -6, C_{32} = -(-3 - 0) = 3$

$C_{21} = -(-8 + 4) = 4, C_{33} = 2 - 0 = 2$

$C_{22} = 4 - 6 = -2, C_{23} = -(-2 + 6) = -4$

$$\text{adj}(A) = \begin{bmatrix} 2 & -9 & -6 \\ 4 & -2 & -4 \\ 2 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 2 \\ -9 & -2 & 3 \\ -6 & -4 & 2 \end{bmatrix}$$

$$\text{Now, } A \cdot \text{adj}(A) = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 4 & 2 \\ -9 & -2 & 3 \\ -6 & -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Alternative : $A^{-1} = \dfrac{\text{adj}(A)}{|A|}$

$$\Rightarrow \quad A\,A^{-1} = \frac{A \cdot [\text{adj}(A)]}{|A|}$$

$$\Rightarrow \quad I|A| = A \cdot [\text{adj}(A)]$$

$$|A| = 8$$

$$\therefore \quad A[\text{adj}(A)] = 8I = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

19. We have, $f(x) = |x|$

$$f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$

So, the function $f^{-1}(x)$ does not exist.

20. Given, $\begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix}$

$$= x[x^2 - q^2] - p[px - pq] + q[pq - px]$$

$$= x^3 - xq^2 - p^2x + p^2q + pq^2 - pqx$$

$$= x(x - q)(x + q) - p^2(x - q) - pq(x - q)$$

$$= (x - q)[x^2 + xq - p^2 - pq]$$

$$= (x - q)(x - p)(x + p + q)$$

21. Let $(a + ib) = i^i$

Taking log on both sides, we get

$$\log(a + ib) = i \log i$$

$$\Rightarrow \quad \log(a + ib) = i(i\,\pi/2)$$

$$\Rightarrow \quad \log(a + ib) = -\frac{\pi}{2}$$

$$\Rightarrow \quad (a + ib) = e^{-\pi/2}$$

On comparing imaginary part of i^i is 0.

22. Given, $(1 + i)^5$

$$= (\sqrt{2})^5\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^5$$

$$= (\sqrt{2})^5\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^5$$

$$= (\sqrt{2})^5\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$$

[by De–Moivre's theorem]

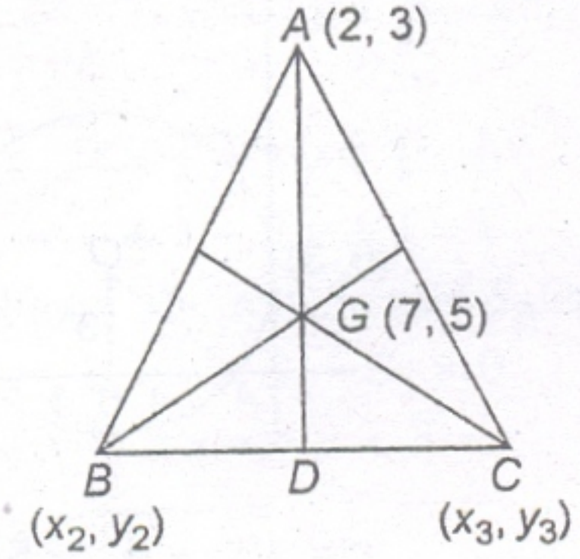
Now, amplitude $= \tan^{-1}\left(\dfrac{y}{x}\right)$

$$= \tan^{-1}\left(\frac{\sin 5\pi/4}{\cos 5\pi/4}\right) = \frac{5\pi}{4}$$

23. Since, D is the mid point of BC. So, coordinate of BC are $\left(\dfrac{x_2 + x_3}{2}, \dfrac{y_2 + y_3}{2}\right)$

Given, $G(7, 5)$ is the centroid of ΔABC

$$\therefore 7 = \frac{2 + x_2 + x_3}{3} \text{ and } 5 = \frac{3 + y_2 + y_3}{3}$$



$\Rightarrow x_2 + x_3 = 21 - 2$ and $y_2 + y_3 = 15 - 3$

$\Rightarrow x_2 + x_3 = 19$ and $y_2 + y_3 = 12$

$\Rightarrow \frac{x_2 + x_3}{2} = \frac{19}{2}$ and $\frac{y_2 + y_3}{2} = 6$

$\therefore$ Coordinate of D are $\left(\frac{19}{2}, 6\right)$.

24. We have, $\lim_{x \to 1} \frac{\tan(x^2 - 1)}{x - 1}$ $\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to 1} \frac{\sec^2(x^2 - 1) \cdot 2x}{1}$$

[using L'Hospital's rule]

$$= 2 \cdot \sec^2(0) = 2$$

25. Given, $y = 2^{\log x}$

$$\Rightarrow \frac{dy}{dx} = 2^{\log x} \cdot \log_e 2 \cdot \frac{1}{x}$$

$$\left[\because \frac{d}{dx}(a^x) = a^x \log_e a\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^{\log x} \cdot \log_e 2}{x}$$

26. Given, $7^{2\log_7 5}$

$= 7^{\log_7 (5)^2}$ $[\because a^{\log_a x} = x, x > 0, x \neq 0, 1]$

$= (5)^2 = 25$

27. Given, $G = \{3, 6, 9, 12\}$

The composition table is

$\times_5$	3	6	9	12
3	9	3	12	6
6	3	6	9	12
9	12	9	6	3
12	6	12	3	9

From the table 6 is the identity element.

28. Since, identity element is its own inverse. So, minimum number of element is 1.

29. We have, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

On squaring both sides, we get

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$$

$$\Rightarrow 4\vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

$\Rightarrow \vec{a}$ and $\vec{b}$ are perpendicular to each other.

So, angle between them is 90°.

Alternative : $\because |\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$\therefore \vec{a}$ and $\vec{b}$ are perpendicular to each other.

So, angle between $\vec{a}$ and $\vec{b}$ is 90°.

30. Given, $\frac{3x^2 + 1}{x^2 - 6x + 8}$

On dividing, we get

$$\frac{3x^2 + 1}{x^2 - 6x + 8} = 3 + \frac{18x - 23}{x^2 - 6x + 8} \quad \ldots(i)$$

Now, $\frac{18x - 23}{(x - 2)(x - 4)} = \frac{A}{x - 2} + \frac{B}{x - 4}$

$\Rightarrow 18x - 23 = A(x - 4) + B(x - 2)$

$\Rightarrow 18x - 23 = (A + B)x - 4A - 2B$

Equating the coefficient of x and constant term, we get

$A + B = 18$

$-4A - 2B = -23$

On solving these equations, we get

$A = -\frac{13}{2}, B = \frac{49}{2}$

$\therefore \frac{18x - 23}{(x - 2)(x - 4)} = -\frac{13}{2(x - 2)} + \frac{49}{2(x - 4)}$

Then, from Eq. (i), we get

$$\frac{3x^2 + 1}{x^2 - 6x + 8} = 3 - \frac{13}{2(x - 2)} + \frac{49}{2(x - 4)}$$

31. Given, equation of circles are

$x^2 + y^2 = 4$, whose radius = 2, centre = (0, 0)

and $x^2 + y^2 - 6x - 8y - 24 = 0$,

whose radius = $\sqrt{9 + 16 - 24} = \sqrt{1} = 1$

and centre = (3, 4)

Now, $c_1\ c_2 = \sqrt{(3-0)^2 + (4-0)^2}$

$= \sqrt{9+16} = 5$

$a_1 + a_2 = 2 + 1 = 3$

Since, $c_1\ c_2 > a_1 + a_2$

$\therefore$ Number of common tangents = 4

32. Given, line is $3x + y + k = 0$

$\Rightarrow \quad y = -3x - k$

And equation of circle is $x^2 + y^2 = 10$

Here, $a^2 = 10,\ m = -3,\ c = -k$

If given line touches the circle, then

Length of intercept = 0

$$\Rightarrow \quad 2\sqrt{\frac{a^2(1+m^2) - c^2}{1+m^2}} = 0$$

$$\Rightarrow \quad 2\sqrt{\frac{10\,(1+9) - k^2}{1+9}} = 0$$

$$\Rightarrow \quad \sqrt{100 - k^2} = 0$$

$$\Rightarrow \quad 100 - k^2 = 0$$

$$\Rightarrow \quad k = \pm 10$$

Alternative : If the given line is tangent to the circle, then the length of the perpendicular from the centre upon the line is equal to the radius of the circle.

ie, $$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right| = r$$

$$\Rightarrow \quad \left|\frac{3\times 0 + 6\times 0 + k}{\sqrt{(3)^2 + (1)^2}}\right| = \sqrt{10}$$

$$\Rightarrow \quad \left|\frac{k}{\sqrt{10}}\right| = \sqrt{10}$$

$$\Rightarrow \quad k = \sqrt{100}$$

$$\Rightarrow \quad k = \pm 10$$

33. Let $p = 2$ is prime

$q = 3$ is odd

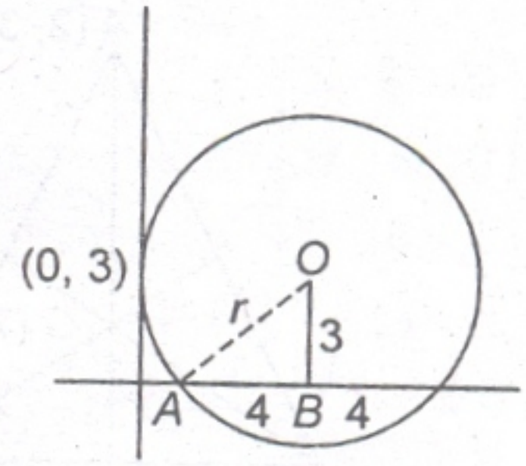
$p \to q = 2$ is prime, then 3 is odd.

Negation of $P \to q$ is $\sim(p \to q)$

$\Rightarrow p \wedge \sim q$

$\Rightarrow$ 2 is prime and 3 is not odd.

34. Given, equation of circle touch the y-axis at (0, 3). So, from figure



$$r = \sqrt{3^2 + 4^2} = 5$$

The point (0, 3) and radius 5 satisfies the equation $x^2 + y^2 \pm 10x - 6y + 9 = 0$.

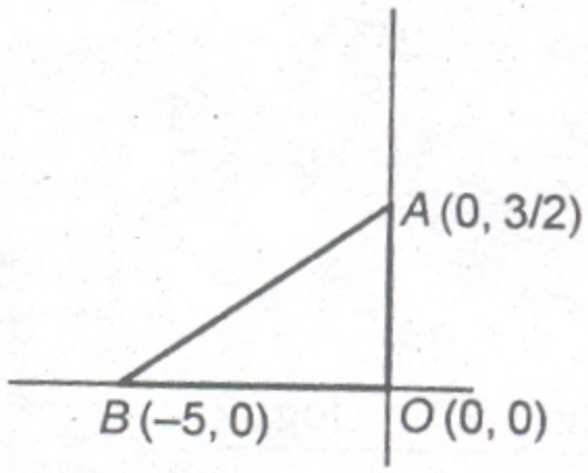
Hence, it is the required equation of circle.

35. Let, $\Delta\ AOB$ is the given triangle

Slope of $AB = \dfrac{\frac{3}{2} - 0}{0 + 5} = \dfrac{3}{10}$

Slope of $BO = \dfrac{0-0}{0+5} = 0$

The equation of line passing through A and perpendicular to BO is $y - 0 = -0\left(x - \dfrac{3}{2}\right)$



$\Rightarrow \quad y = 0$ ...(i)

and equation of line passing through 0 and perpendicular to AB is $y - 0 = -\dfrac{10}{3}(x - 0)$

$\Rightarrow \quad y = -\dfrac{10}{3}x$ ...(ii)

The intersection point of Eqs. (i) and (ii) is (0, 0), which is the required orthocentre.

36. Given, equation of circles are

$S_1 \equiv x^2 + y^2 - 6x - 6y + 4 = 0$

$S_2 \equiv x^2 + y^2 - 2x - 4y + 3 = 0$

and $S_3 \equiv x^2 + y^2 + 2kx + 2y + 1 = 0$

Now, radical axis of circle S_1 and S_2 is

$S_1 - S_2 = 0$

$$\Rightarrow x^2 + y^2 - 6x - 6y + 4 - x^2 - y^2 + 2x + 4y - 3 = 0$$

$$\Rightarrow -4x - 2y + 1 = 0$$

$$\Rightarrow 4x + 2y + 1 = 0 \quad \ldots(i)$$

Radical axis of circle S_2 and S_3 is

$$S_2 - S_3 = 0$$

$$\Rightarrow x^2 + y^2 - 2x - 4y + 3 - x^2 - y^2 - 2kx - 2y - 1 = 0$$

$$\Rightarrow -(2 + 2k)x - 6y + 2 = 0$$

$$\Rightarrow (2 + 2k)x + 6y - 2 = 0 \quad \ldots(ii)$$

For existence of radical centre

$$\begin{vmatrix} 4 & 2 \\ 2 + 2k & 6 \end{vmatrix} \neq 0$$

$$\Rightarrow 24 - 2(2 + 2k) \neq 0$$

$$\Rightarrow 24 - 4 - 4k \neq 0$$

$$\Rightarrow 20 - 4k \neq 0$$

$$\Rightarrow k \neq 5$$

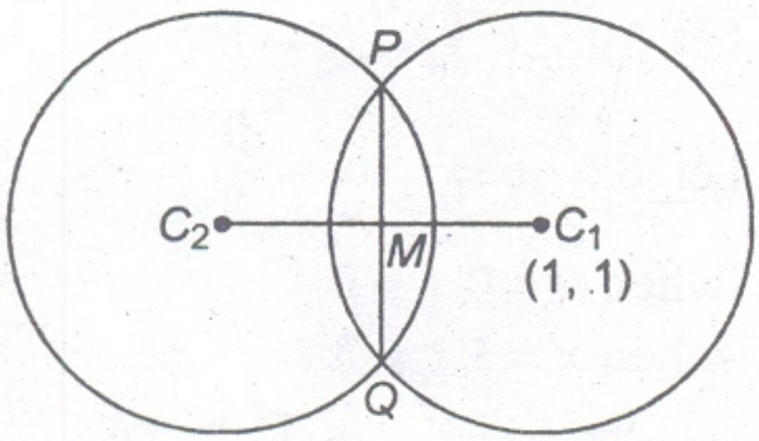
37. Given, circles are

$$S_1 \equiv x^2 + y^2 - 2x - 2y - 7 = 0$$

$$S_2 \equiv x^2 + y^2 + 4x + 2y + k = 0$$

Here, $g_1 = -1, f_1 = -1, c_1 = -7$

$g_2 = 2, f_2 = 1, c_2 = k$



Equation of common chord is $S_1 - S_2 = 0$

$$\Rightarrow x^2 + y^2 - 2x - 2y - 7 - x^2 - y^2 - 4x - 2y - k = 0$$

$$\Rightarrow -6x - 4y - 7 - k = 0$$

$$\Rightarrow 6x + 4y + 7 + k = 0 \quad \ldots(i)$$

Since, S_1 and S_2 cut orthogonally

$$\therefore 2(g_1g_2 + f_1f_2) = c_1 + c_2$$

$$\Rightarrow 2(-2 - 1) = -7 + k$$

$$\Rightarrow -6 + 7 = k$$

$$\Rightarrow k = 1$$

Then, from Eqs. (i), we get

$$6x + 4y + 8 = 0$$

Now, length of the common chord

$$r_1 = \sqrt{1 + 1 + 7} = 3$$

$$c_1 = (1, 1)$$

Let C_1M = perpendicular distance from centre $C_1(1, 1)$ to the common chord

$$6x + 4y + 8 = 0$$

$$\Rightarrow C_1M = \frac{|6 + 4 + 8|}{\sqrt{6^2 + 4^2}} = \frac{|18|}{\sqrt{5^2}} = \frac{18}{2\sqrt{13}} = \frac{9}{\sqrt{13}}$$

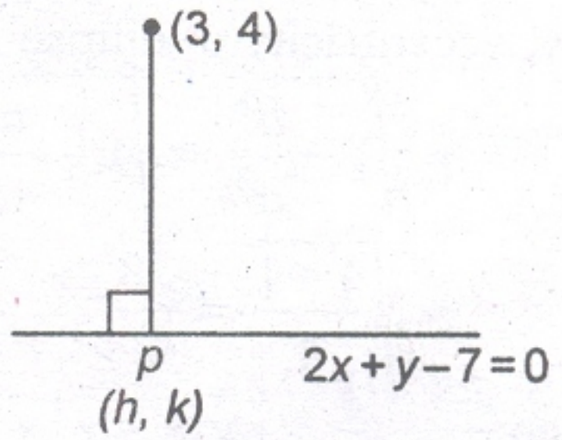
Now, $PQ = 2PM = 2\sqrt{(C_1P)^2 - (C_1M)^2}$

$$= 2\sqrt{9 - \left(\frac{9}{\sqrt{13}}\right)^2}$$

$$= 2\sqrt{9 - \frac{81}{13}}$$

$$= 2\sqrt{\frac{36}{13}} = \frac{12}{\sqrt{13}}$$

38. We know that foot of the perpendicular (h, k) from (x_1, y_1) to the line $ax + by + c = 0$ is given by



$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2} \quad \ldots(i)$$

Here, point $(x_1, y_1) = (3, 4)$

and $ax + by + c = 2x + y - 7 = 0$

$$\therefore a = 2, b = 1, c = -7$$

Then, from Eq. (i)

$$\frac{h - 3}{2} = \frac{k - 4}{1} = \frac{-(2 \times 3 + 1 \times 4 - 7)}{2^2 + 1^2}$$

$$\Rightarrow \frac{h - 3}{2} = \frac{k - 4}{1} = \frac{-3}{5}$$

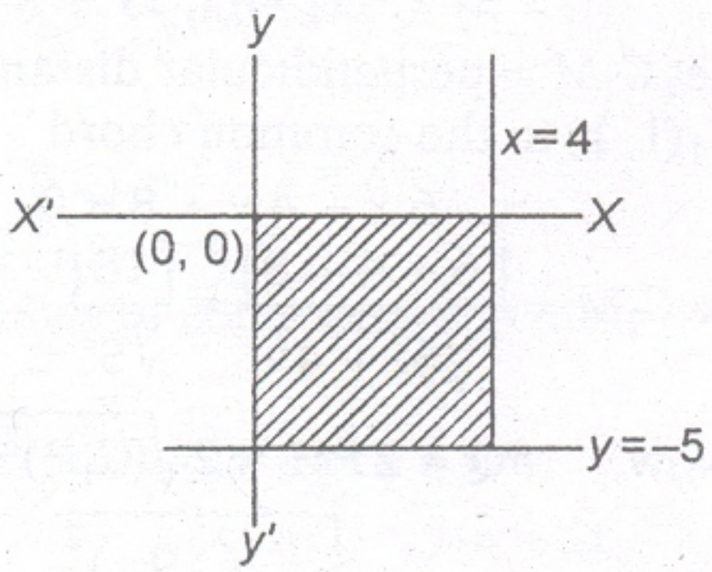
$$\Rightarrow \frac{h - 3}{2} = \frac{-3}{5} \text{ and } \frac{k - 4}{1} = \frac{-3}{5}$$

$$\Rightarrow h = \frac{-6}{5} + 3 \text{ and } k = \frac{-3}{5} + 4$$

$$\Rightarrow h = \frac{+9}{5} \text{ and } k = \frac{17}{5}$$

39. Given, pair of lines $xy = 0$, $x - 4 = 0 \Rightarrow x = 4$ and $y + 5 = 0$.

The figure is a rectangle.



In which length is 5 unit and breadth is 4 unit
So, area = $4 \times 5 = 20$ sq unit.

40. Given, ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose area is $2\pi ab$. The auxillary circle of the ellipse is $x^2 + y^2 = a^2$ whose area is πa^2.

Given that $\pi a^2 = 2\pi ab$

$$a = 2b$$

Now, eccentricity of ellipse

$$= \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{b^2}{4b^2}}$$

$$= \sqrt{\frac{4-1}{4}} = \frac{\sqrt{3}}{2}$$

41. Given, $y = -x^2 + 6x - 3$

$$\Rightarrow \quad \frac{dy}{dx} = -2x + 6 = y'$$

y is increasing, if $y' > 0$

$$\Rightarrow \quad -2x + 6 > 0$$
$$\Rightarrow \quad -2x > -6$$
$$\Rightarrow \quad 2x > 6$$
$$\Rightarrow \quad x > 3$$

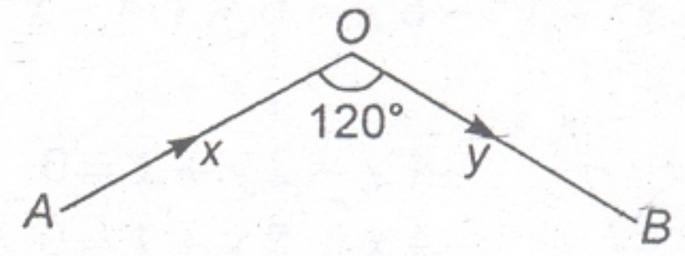
42. Let $I = \int_0^{\pi/2} (\sin^{100} x - \cos^{100} x)\, dx$

$$= \int_0^{\pi/2} \sin^{100} x\, dx - \int_0^{\pi/2} \cos^{100} x\, dx$$

$$= \left[\frac{(\sin x)^{101}}{101} \cdot \cos x\right]_0^{\pi/2} - \left[\frac{(\cos x)^{101}}{101}(-\sin x)\right]_0^{\pi/2}$$

$$= 0 + 0$$
$$= 0$$

43. Given, speed of x is 4 km/h and y is 3 km/h.
After time t the distance covered by x is $4t$ and y is $3t$.



Let shortest distance between x and $y = A$.
Then by cosine law

$$A^2 = (4t)^2 + (3t)^2 - (4t)(3t)\, 2\cos 120°$$

$$\Rightarrow A^2 = 16t^2 + 9t^2 - 24t^2\left(-\frac{1}{2}\right)$$

$$\Rightarrow A^2 = 25t^2 + 12t^2$$

$$\Rightarrow A^2 = 37\, t^2 \qquad \ldots(i)$$

$$\Rightarrow \quad A = \sqrt{37}\, t$$

If $\quad t = 1$ h, then $A = \sqrt{37}$ km

Now, differentiating Eq. (i) w.r.t. t, we get

$$2AA' = 37\,(2t)$$

After $t = 1$ h, we get

$$2\sqrt{37}\,A' = 2(37)$$

$$\Rightarrow \quad A' = \sqrt{37}$$

Thus, rate at which shortest distance A changes with time is $\sqrt{37}$ km/h.

44. Given, $k\int_0^1 x \cdot f(3x)\, dx = \int_0^3 t \cdot f(t)\, dt \qquad \ldots(i)$

Let $\quad I = k\int_0^1 x\, f(3x)\, dx$

Let $3x = t \Rightarrow \quad dx = \frac{dt}{3}$

when $x = 0, t = 0$
when $x = 1, t = 3$

$$\therefore \quad I = k\int_0^3 \frac{t}{3} \cdot f(t) \cdot \frac{dt}{3}$$

$$= \frac{k}{9}\int_0^3 t\, f(t)\, dt$$

Now, from Eq. (i), we get

$$\frac{k}{9}\int_0^3 t \cdot f(t)\, dt = \int_0^3 t \cdot f(t)\, dt$$

$$\Rightarrow \quad \frac{k}{9} = 1 \Rightarrow \quad k = 9$$

45. Let $I = \int \frac{1}{1 + \cos 8x}\, dx$

$$= \int \frac{1}{2\cos^2 4x}\, dx$$

$$= \frac{1}{2}\int \sec^2 4x\, dx$$

$$= \frac{1}{2}\frac{\tan 4x}{4} + c$$

$$= \frac{\tan 4x}{8} + c$$

46. Given, $\sec^{-1}\left(\frac{1+x}{1-y}\right) = a$

$$\Rightarrow \quad \frac{1+x}{1-y} = \sec a$$

$$\Rightarrow \quad 1 + x = (1 - y)\sec a$$

$$\Rightarrow \quad y \sec a = \sec a - 1 - x$$

$$\Rightarrow \quad \frac{dy}{dx}\sec a = -1$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-1}{\sec a} = \frac{-1}{\left(\frac{1+x}{1-y}\right)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-(1-y)}{(1+x)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y-1}{x+1}$$

47. Given, $y = \cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}$

$$\Rightarrow \quad y = \cos^2\frac{3x}{2} - \left(1 - \cos^2\frac{3x}{2}\right)$$

$$\Rightarrow \quad y = 2\cos^2\frac{3x}{2} - 1$$

$$\Rightarrow \frac{dy}{dx} = 2 - 2\cos\frac{3x}{2}\left(-\sin\frac{3x}{2}\right)\left(\frac{3}{2}\right)$$

$$\Rightarrow \frac{dy}{dx} = -6\cos\frac{3x}{2}\sin\frac{3x}{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -6\left[\cos\frac{3x}{2}\left(\cos\frac{3x}{2}\right)\cdot\frac{3}{2} - \sin\frac{3x}{2}\sin\frac{3x}{2}\cdot\frac{3}{2}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -9\left[\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -9y$$

48. Given, $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$

Since, x is continuous

$\therefore \quad \lim_{x\to 0} f(x) = f(0)$

$$\Rightarrow \quad \lim_{x\to 0}\frac{1-\cos x}{x^2} = k$$

$$\Rightarrow \lim_{x\to 0}\frac{-(-\sin x)}{2x} = k \text{ [using L'Hospital's rule]}$$

$$\Rightarrow \quad \frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x} = k$$

$$\Rightarrow \quad \frac{1}{2}\cdot 1 = k \Rightarrow k = \frac{1}{2}$$

49. Given, $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$= (1+\omega)(-\omega)(1+\omega^3\cdot\omega)\{1+(\omega^3)^2\cdot\omega^2\}$

$[\because 1+\omega+\omega^2 = 0]$

$= (1+\omega)(-\omega)(1+\omega)(1+\omega^2) \qquad [\because \omega^3 = 1]$

$= (1+\omega)^2(-\omega-\omega^3)$

$= (1+\omega^2+2\omega)(-\omega-1)$

$= (-\omega+2\omega)(+\omega^2) = \omega^3 = 1$

50. Given, $x^x = y^y$

Taking log on both sides, we get

$x \log x = y \log y$

Differentiating w.r.t. y, we get

$$y\cdot\frac{1}{y}\cdot\frac{dy}{dx} + \log y\frac{dy}{dx} = x\frac{1}{x} + \log x$$

$$\Rightarrow \quad \frac{dy}{dx}(1+\log y) = 1 + \log x$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1+\log x}{1+\log y}$$

51. Let $I = \int e^x(x^5 + 5x^4 + 1)\,dx$

$$= \int e^x x^5 dx + 5\int e^x x^4 dx + \int e^x dx$$

$$= x^5e^x - \int 5x^4e^x\,dx + 5\int e^x x^4\,dx + e^x$$

$$= x^5e^x + e^x + c = e^x(x^5+1) + c$$

52. Let $I = \int\frac{x^2+1}{x^2-1}\,dx$

$$\Rightarrow \quad I = \int\frac{x^2+1-1+1}{x^2-1}\,dx$$

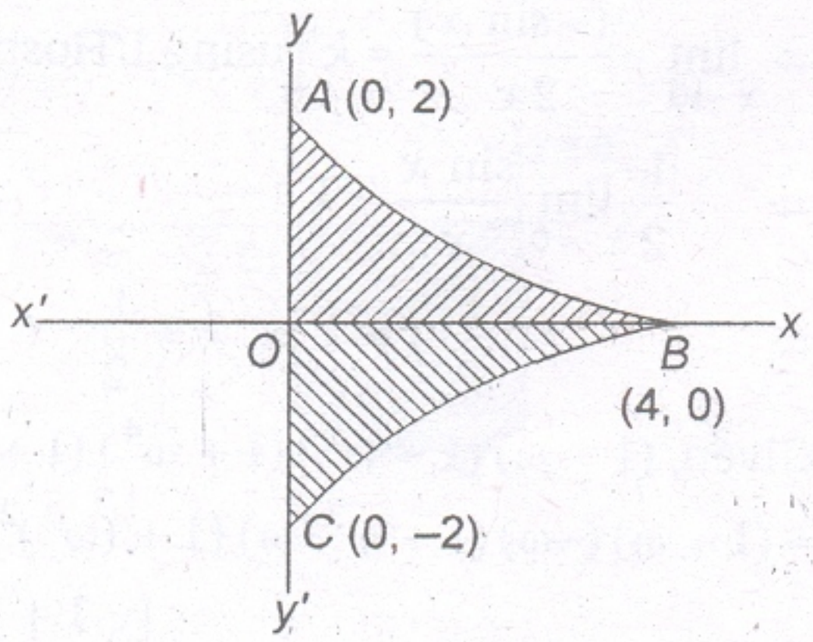
$$\Rightarrow \quad I = \int\frac{x^2-1}{x^2-1}\,dx + \int\frac{2}{x^2-1}\,dx$$

$$\Rightarrow \quad I = \int 1\,dx + 2\int\frac{1}{x^2-1}\,dx$$

$$\Rightarrow \quad I = x + 2\cdot\frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + c$$

$$\Rightarrow \quad I = x + \log\left(\frac{x-1}{x+1}\right) + c$$

53. Given, curve $x = 4 - y^2$ and y-axis.



The formed area is $ABCOA$

$= 2 \times$ area of $ABOA$

$$= 2 \times \int_0^4 \sqrt{4 - x}\, dx$$

$$= 2\left[-\frac{(4 - x)^{3/2}}{3/2} \right]_0^4$$

$$= 2\left[-\frac{2}{3} \times 0 + \frac{2}{3}(4)^{3/2} \right]$$

$$= \frac{4}{3} \times 8 = \frac{32}{3} \text{ sq unit}$$

54. Equation of line whose slope is equal to y intercept, is

$$y = cx + c = c(x + 1) \quad \ldots(i)$$

$$\Rightarrow \quad \frac{dy}{dx} = c \quad \ldots(ii)$$

On putting the value of c from Eq. (i) in Eq. (ii), we get

$$\frac{dy}{dx} = \frac{y}{x + 1}$$

$$\Rightarrow \quad (x + 1)\frac{dy}{dx} - y = 0$$

Which is the required differential equation.

55. Given differential equation is

$$\left[1 + \left(\frac{dy}{dx}\right)^5\right]^{1/3} = \frac{d^2y}{dx^2}$$

On cubing both sides, we get

$$1 + \left(\frac{dy}{dx}\right)^5 = \left(\frac{d^2y}{dx^2}\right)^3$$

Now, order is 2 and degree is 3.

56. Given, $y^2 = x \quad \ldots(i)$

$$\Rightarrow \quad 2y\frac{dy}{dx} = 1$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2y} = \text{slope}$$

Also given, $\theta = 45°$

$\therefore$ Slope $\tan 45° = 1$

$$\Rightarrow \quad \frac{1}{2y} = 1$$

$$\Rightarrow \quad y = \frac{1}{2}$$

From Eq. (i), if $y = 1/2$, then $x = \frac{1}{4}$

$\therefore$ Required point is $\left(\frac{1}{4}, \frac{1}{2}\right)$.

57. Given, curve $x^2y^2 = a^4$

$$\Rightarrow \quad y^2 = \frac{a^4}{x^2}$$

On differentiating, we get

$$2y\frac{dy}{dx} = \frac{-2a^4}{x^3}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-a^4}{x^3y}$$

at $(-a, a)$, $\frac{dy}{dx} = \frac{-a^4}{-a^3 \cdot a} = 1$

Now, length of subtangent to the given curve at $(-a, a)$ is

$$\frac{y}{dy/dx} = \frac{a}{1} = a$$

58. We know that, if $a = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \ldots$

Then, the total number of positive divisors of a is $T(a) = (\alpha_1 + 1)(\alpha_2 + 1) \ldots\ldots$

Given, $252 = 2^2 \times 3^2 \times 7^1$

Here, $\alpha_1 = 2, \alpha_2 = 2, \alpha_3 = 1$

$\therefore \quad T(a) = (2 + 1)(2 + 1)(1 + 1)$

$= 3 \cdot 3 \cdot 2$

$= 18$

59. We have, $5^{124} = (5^3)^{41} \cdot 5$

Now, $5^3 = 1 \pmod{124}$

$\therefore \quad (5^3)^{41} = 1 \pmod{124}$

$(5^3)^{41} \cdot 5 = 1 \cdot 5 \pmod{124}$

$\Rightarrow \quad 5^{124} = 5 \pmod{124}$

60. The set of odd integers under addition is not a group. Since, under addition 0 is identity element which is not an odd number.