ANSWERS

PHYSICS

1. (d)	2. (c)	3. (c)	4. (b)	5. (d)	6. (c)	7. (d)	8. (c)	9. (d)	10. (a)
11. (d)	12. (d)	13. (c)	14. (a)	15. (b)	16. (c)	17. (d)	18. (c)	19. (b)	20. (c)
21. (d)	22. (a)	23. (b)	24. (c)	25. (b)	26. (a)	27. (b)	28. (c)	29. (a)	30. (c)
31. (b)	32. (c)	33. (d)	34. (b)	35. (c)	36. (a)	37. (c)	38. (a)	39. (c)	40. (d)
41. (c)	42. (b)	43. (c)	44. (c)	45. (a)	46. (a)	47. (b)	48. (b)	49. (d)	50. (a)
51. (d)	52. (a)	53. (c)	54. (b)	55. (a)	56. (c)	57. (c)	58. (d)	59. (a)	60. (d)

CHEMISTRY

1. (c)	2. (a)	3. (b)	4. (a)	5. (d)	6. (a)	7. (c)	8. (b)	9. (c)	10. (a)
11. (a)	12. (d)	13. (c)	14. (c)	15. (c)	16. (a,b,d)	17. (a)	18. (a,d)	19. (a,c)	20. (b)
21. (c)	22. (a)	23. (a)	24. (a)	25. (c)	26. (d)	27. (b)	28. (b)	29. (b)	30. (c)
31. (d)	32. (c)	33. (b)	34. (c)	35. (a)	36. (a)	37. (b)	38. (c)	39. (a)	40. (a)
41. (c)	42. (c)	43. (d)	44. (c)	45. (c)	46. (b)	47. (b)	48. (a)	49. (c)	50. (c)
51. (b)	52. (b)	53. (b)	54. (c)	55. (a)	56. (a)	57. (c)	58. (a)	59. (c)	60. (a)

MATHEMATICS

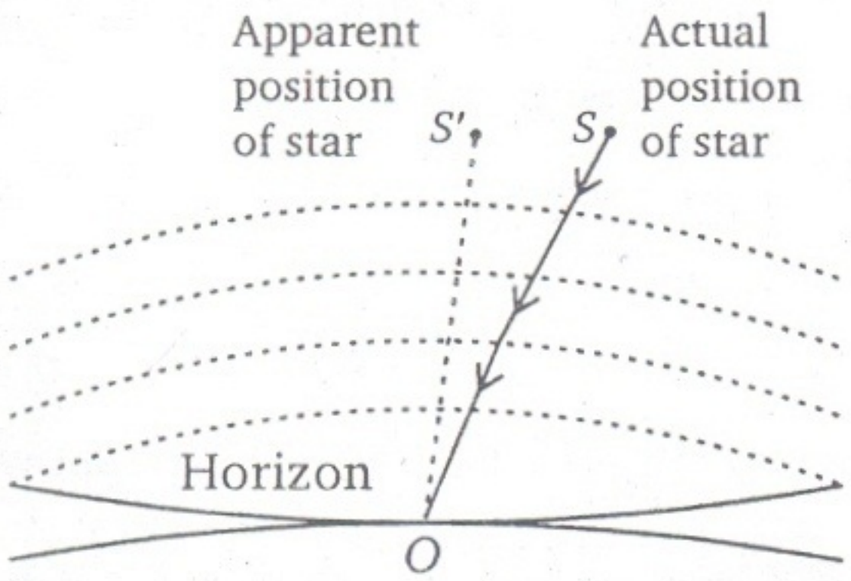
1. (a)	2. (d)	3. (b)	4. (a)	5. (c)	6. (d)	7. (a)	8. (d)	9. (a)	10. (c)
11. (d)	12. (b)	13. (c)	14. (a)	15. (a)	16. (b)	17. (d)	18. (c)	19. (d)	20. (a)
21. (b)	22. (a)	23. (c)	24. (d)	25. (b)	26. (a)	27. (c)	28. (d)	29. (b)	30. (c)
31. (b)	32. (c)	33. (a)	34. (d)	35. (c)	36. (b)	37. (b)	38. (a)	39. (b)	40. (c)
41. (b)	42. (c)	43. (d)	44. (a)	45. (b)	46. (c)	47. (d)	48. (b)	49. (a)	50. (b)
51. (c)	52. (b)	53. (c)	54. (d)	55. (a)	56. (d)	57. (a)	58. (c)	59. (b)	60. (c)

HINTS AND SOLUTIONS

Physics

1. The atmosphere can be considered to consist of a number of parallel layers of air of different densities and therefore of different refractive indices. The density and the refractive index of layers decrease with altitude.

The rays of light coming from a star to the earth are thus continually refracted from the



rarer to the denser layers and so they bend slightly towards the normal at each refraction from one layer to the next. Thus, they follow a curved path and reach the eyes of the observer at O as shown in figure. Hence, the image of the star S is seen as S'. But due to the wind and the convection currents in air the density of layers keep on changing and hence, the position of the star S' as seen, keeps on changing. These different images of the star give an impression to an observer that the star is twinkling.

2. Width of the diffraction band is given by

$$\beta = \frac{\lambda D}{d}$$

where D= distance between slit and the screen

λ = wavelength of light used and d= width of slit.

Hence, width of the diffraction band varies directly as the distance between the slit and the screen.

3. According to Malus' law

$$I = I_0 \cos^2 \theta$$

$$= I_0(\cos^2 60°)$$

$$= I_0 \times \left(\frac{1}{2}\right)^2$$

$$= \frac{I_0}{4}$$

4. In figure (b), the particle is slowly accelerated first and reaches a constant velocity for the straight line portion. Then velocity decreases are finally it stops when it reaches the top straight line portion.

Hence, figure (b) represents one dimensional motion of a particle.

5. Since, the initial position of cyclist coincides with final position, so his net displacement is zero.

$$\text{Average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$= \frac{OP + PQ + QO}{10} \text{ km/min.}$$

$$= \frac{1 + \frac{\pi}{2} \times 1 + 1}{10} \text{ km/min}$$

$$= \frac{\pi + 4}{20} \times 60 \text{ km/h}$$

$$= 21.4 \text{ km/h}$$

6. When a low flying aircraft passing overhead, then it causes a change in magnetic flux and thus, causes a disturbance in picture on our TV screen.

7. $\beta = \frac{\lambda D}{d} = \frac{600 \times 10^{-9} \times 2}{1 \times 10^{-3}} = 12 \times 10^{-4} \text{ m}$

So, distance between the first dark fringes on either side of the central bright fringe

$$X = 2\beta$$

$$= 2 \times 12 \times 10^{-4} \text{ m}$$

$$= 24 \times 10^{-4} \text{ m}$$

$$= 2.4 \text{ mm}$$

8. Resistivity, $\rho = \frac{m}{ne^2\tau}$

$$\because \quad [\rho] = \frac{[M]}{[L^{-3}][AT]^2[T]}$$

$$= [ML^3 A^{-2} T^{-3}]$$

So, electrical conductivity.

$$\sigma = \frac{1}{\rho}$$

$$[\sigma] = \frac{1}{[\rho]} = [M^{-1}L^{-3}A^2T^3]$$

9. Terminal voltage of the battery after closing the circuit is

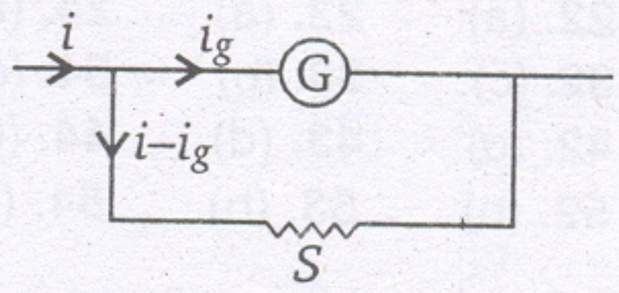
$$V = E - ir$$

$$= 10 - 0.5 \times 3$$

$$= 10 - 1.5$$

$$= 8.5 \text{ V}$$

10. To convert a galvanometer into an ammeter, a shunt resistance in parallel is connected to galvanometer.



Since, galvanometer G and shunt S are in parallel, hence $i_g G = (i - ig)$

$$\Rightarrow \quad S = \frac{i_g G}{i - i_g}$$

$$= \frac{4 \times 10^{-3} \times 15}{6 - 4 \times 10^{-3}}$$

$$= \frac{60 \times 10^{-3}}{5.996}$$

$$= 10 \times 10^{-3} \Omega$$

$$= 10 \text{ m } \Omega$$

11. In a conductor, the electron number density *i.e.*, number of electrons per unit volume of a conductor is very large ($\approx 10^{28} \text{ m}^{-3}$), so large current in a conductor is obtained irrespective of their small drift speed.

12. Megnetic force on straight wire $F = Bil \sin\theta = Bil \sin 90^\circ = Bil$

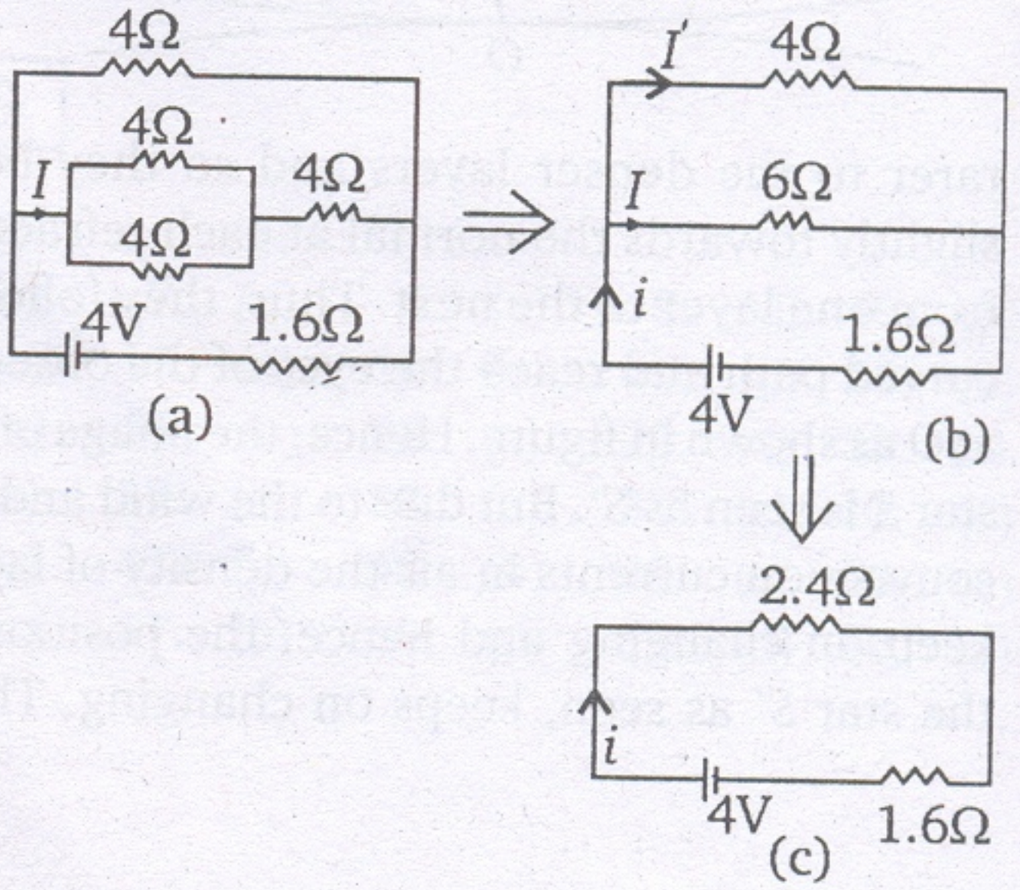
For equilibrium of wire in mid-air,

$$F = mg$$

$$Bil = mg$$

$$\therefore \quad B = \frac{mg}{il} = \frac{200 \times 10^{-3} \times 9.8}{2 \times 1.5} = 0.65 \text{ T}$$

13. We can simplify the network as shown :



So, net resistance,

$$R = 2.4 + 1.6 = 4.0\Omega$$

Therefore, current from the battery,

$$i = \frac{V}{R} = \frac{4}{4} = 1\,\text{A}$$

Now from the circuit (b),

$$4I' = 6I$$

$$\Rightarrow \quad I' = \frac{3}{2}I$$

$$\text{but } i = I + I' = I + \frac{3}{2}I = \frac{5}{2}I$$

$$\therefore \quad 1 = \frac{5}{2}I$$

$$\Rightarrow \quad I = \frac{2}{5} = 0.4\,\text{A}$$

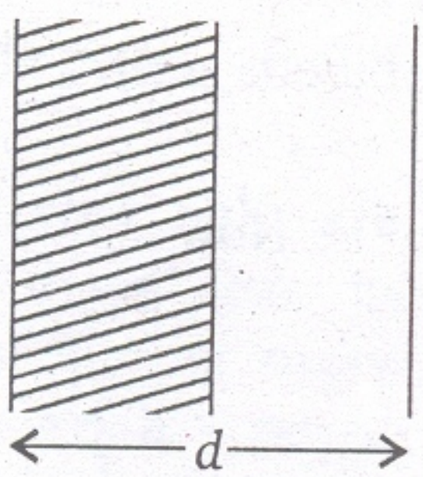
14. Total electric flux through enclosed surface

$$\phi = \frac{q}{\varepsilon_0} + \left(\frac{-q}{\varepsilon_0}\right) = 0$$

Note : As Gaussian surface is not a enclosed surface, hence the question should have been 'an enclosed surface' and not a Gaussian surface.

15. Initial capacitance

$$C = \frac{\varepsilon_0 A}{d}$$



When it is half filled by a dielectric of dielectric constant k, then

$$C_1 = \frac{K\varepsilon_0 A}{d/2} = 2K\frac{\varepsilon_0 A}{d}$$

and $\quad C_2 = \frac{\varepsilon_o A}{d/2} = \frac{2\varepsilon_0 A}{d}$

$$\therefore \quad \frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K} + 1\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{1}{5} + 1\right)$$

$$= \frac{6}{10}\frac{d}{\varepsilon_0 A}$$

$$\therefore \quad C' = \frac{5\varepsilon_0 A}{3d}$$

Hence, increase in capacitance

$$= \frac{\frac{5}{3}\frac{\varepsilon_0 A}{d} - \frac{\varepsilon_0 A}{d}}{\frac{\varepsilon_0 A}{d}}$$

$$= \frac{5}{3} - 1 = \frac{2}{3}$$

$$= 66.6\%$$

16. When a comb runs through one's dry hair, then comb gets charged and when it comes close to paper, it induces opposite charges in paper. The field due to the charges in comb, polarises the atoms in the paper. Finally it attracts the paper.

17. Dry atmosphere is not a conductor. Only when top of the atmosphere increases to a very large value and there is moisture in the air the lightning strikes the ground. Since 400 kV is not very high voltage so, it will not strike the ground.

18. Specific charge on proton $= \left(\frac{e}{m}\right)_p$

$= 9.6 \times 10^7$ C/kg specific charge on α-particle,

$$\left(\frac{q}{m}\right)_\alpha = \frac{2e}{4m} = \frac{1}{2}\left(\frac{e}{m}\right)_p = \frac{1}{2} \times 9.6 \times 10^7$$

$$= 4.8 \times 10^7 \text{ C/kg}$$

19. Work function is given by

$$\phi = \frac{hc}{\lambda}$$

or $\quad \phi \propto \frac{1}{\lambda}$

$$\therefore \quad \frac{\phi_1}{\phi_2} = \frac{\lambda_2}{\lambda_1} = \frac{600}{300} = \frac{2}{1}$$

21. For first line of Lyman series,

$n_1 = 1$ and $n_2 = 2$

$$\therefore \quad \frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4}$$

For first line of Paschen series

$n_1 = 3$ and $n_2 = 4$

$$\therefore \quad \frac{1}{\lambda_2} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$= \frac{7R}{144}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{7R}{144} \times \frac{4}{3R} = \frac{7}{108}$$

22. $R = R_0\, e^{-\lambda t}$

$$\Rightarrow \qquad \left(\frac{1}{3}\right) = e^{-\lambda \times 3} = e^{-3\lambda} \qquad \ldots(i)$$

Again, $\dfrac{R'}{R_0} = e^{-\lambda \times 9} = e^{-9\lambda} = (e^{-3\lambda})^3$

$$= \left(\frac{1}{3}\right)^3 \text{ [from Eq. (i)]}$$

$$= \frac{1}{27}$$

$$\Rightarrow \qquad R' = \frac{R_0}{27}$$

Hence, in 9 days activity will become $\left(\dfrac{1}{27}\right)$ of the original value.

23. The output of the circuit is,

$$y = \overline{\bar{A} + \bar{B}}$$

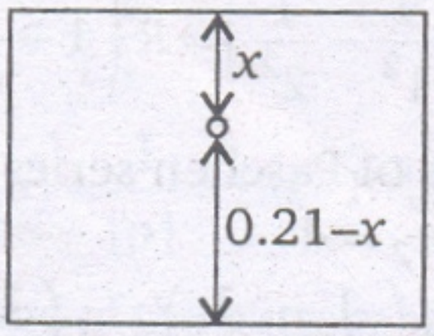
$$= \bar{\bar{A}} \cdot \bar{\bar{B}}$$

$$= A \cdot B \quad (\because \bar{\bar{A}} = A \text{ and } \bar{\bar{B}} = B)$$

which is the output of an AND gate.

25. Due to forward bias at the emitter-base junction, the majority charge carrier electrons of emitter get repelled from the negative terminal and move towards base. Some of these electrons combine with the majority charge carrier holes present in the base and most of the electrons reach the collector, crossing the collector-base junction. This implies that collector current is always less than the emitter current due to the reason (b).

26. Refractive index $= \dfrac{\text{real depth}}{\text{apparent depth}}$



$$\therefore \qquad \frac{x}{0.04} = \frac{0.21 - x}{0.10}$$

$$\Rightarrow \qquad 0.21 \times 0.04 - x \times 0.04 = 0.10x$$

$$\Rightarrow \qquad x = \frac{0.21 \times 0.04}{0.14} = 0.06 \text{ m}$$

27. Angular dispersion,

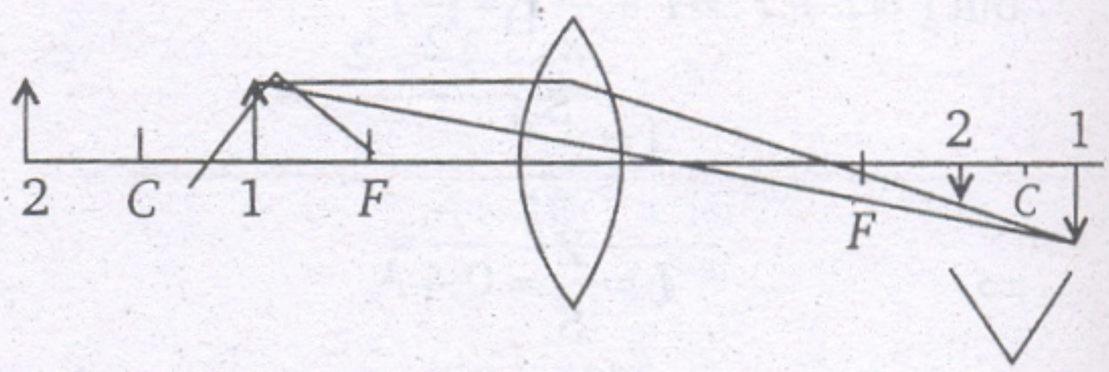
$$\delta_b - \delta_r = (\mu_b - \mu_r)A$$

$$= (1.659 - 1.641)5°$$

$$= 0.09°$$

28. Let as shown, 1 and 2 are positions of objects and images in two different situations.

Object



Image

It is given

$$\left|\frac{v_1}{u_1}\right| = 2\left|\frac{v_2}{u_2}\right|$$

Here, $u_1 = -15$ cm , $u_2 = -20$ cm

$$\therefore \qquad v_1 = 2v_2 \times \frac{u_1}{u_2} = 2v_2 \times \frac{15}{20} = \frac{3}{2} v_2$$

Now $$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\therefore \qquad \frac{1}{f} = \frac{1}{v_1} - \frac{1}{u_1} \text{ and } \frac{1}{f} = \frac{1}{v_2} - \frac{1}{u_2}$$

So, $$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{v_2} - \frac{1}{u_2}$$

$$\Rightarrow \qquad \frac{2}{3v_2} + \frac{1}{15} = \frac{1}{v_2} + \frac{1}{20}$$

$$\Rightarrow \qquad v = 20 \text{ cm}$$

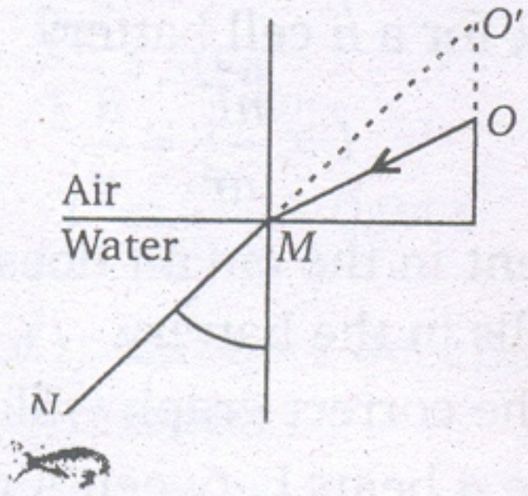
$$\therefore \qquad \frac{v_1}{u_1} = 2\frac{v_2}{u_2} = 2 \times \frac{20}{20} = 2$$

$$\Rightarrow \qquad v_1 = 2u_1 = 2 \times 15 = 30\text{cm}$$

Therefore, $\dfrac{1}{f} = \dfrac{1}{v_1} - \dfrac{1}{u_1} = \dfrac{1}{15} + \dfrac{1}{30} = \dfrac{3}{30}$

$$\therefore \qquad f = 10 \text{ cm} = 0.10 \text{ m}$$

29. When a ray from man O in air (rarer medium) goes to water (denser medium), then it bends towards the normal. Extend MN backwards meet at point O'. Therefore, it appears to the fish as if the man is taller than what he actually is.



Fish

30. For no deviation,

$$(\mu - 1)A + (\mu' - 1)A' = 0$$

$$\Rightarrow \quad A' = -\frac{(\mu - 1)A}{(\mu' - 1)}$$

$$= -\frac{(1.54 - 1)4°}{(1.72 - 1)}$$

$$= -3°$$

Negative sign implies that two prisms should be connected in opposition.

31. The rings observed in reflected light are exactly complementary to those seen in transmitted light. Corresponding to every dark ring in reflected light there is a bright ring in transmitted light. The ray reflected at the upper surface of the air-film suffers no phase change while the ray reflected internally at the lower surface suffers a phase change of π.

32. The strength of solution is given by

$$c = \frac{\theta}{l \times s}$$

where the symbols have their usual meanings.

Here,

$$\theta = 19°, \quad l = 20 \text{ cm} = 0.20 \text{ m},$$

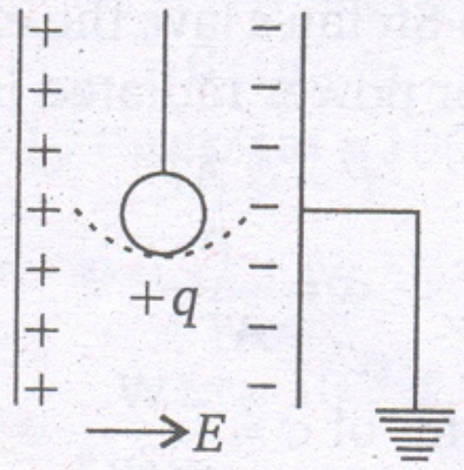
$$S = 0.5 \text{ deg m}^2/\text{kg}$$

$$\therefore \quad c = \frac{19}{0.20 \times 0.5} = 190 \text{ kg-m}^{-3}$$

The sugar sample dissolved in a m^3 of water is 200 kg in which 190 kg is pure sugar. Therefore, purity is

$$\frac{190}{200} \times 100 = 95\%$$

33. Time period of simple pendulum in air



$$T = 2\pi \frac{\sqrt{l}}{g}$$

When it is suspended between vertical plates of a charged parallel plate capacitor, then accelertion due to electric field,

$$a = \frac{qE}{m}$$

This acceleration is acting horizontally and acceleration due to gravity is acting vertically.

So, effective acceleration

$$g' = \sqrt{g^2 + a^2} = \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

Hence, $$T' = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

34. The given arrangement of nine plates is equivalent to the parallel combination of 8 capacitors.

The capacity of each capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{8.854 \times 10^{-12} \times 5 \times 10^{-4}}{0.885 \times 10^{-2}} = 0.5 \text{ pF}$$

Hence, the capacity of 8 capacitors

$$= 8C = 8 \times 0.5 = 4 \text{ pF}$$

35. According to Kepler's third law

$$T^2 \propto R^3$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{R_2}{R_1}\right)^{3/2}$$

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{3R}{R}\right)^{3/2}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \sqrt{27}$$

$$\therefore \quad T_2 = \sqrt{27}\, T_1 = \sqrt{27} \times 4 = 4\sqrt{27}\text{h}$$

37. According to Stefan's law, the energy radiated per second or power radiated is given by

$$P = \sigma A T^4$$

$$\therefore \quad \sigma = \frac{P}{AT^4}$$

Therefore, unit of $\sigma = \frac{\text{W}}{\text{m}^2\text{K}^4} = \text{Wm}^{-2}\text{K}^{-4}$

38. In an adiabatic process,

$$PV^{\gamma} = \text{constant}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \left(\frac{1}{8}\right)^{5/3}$$

$$\Rightarrow \quad \frac{P_1}{P_2} = \left(\frac{1}{2^3}\right)^{5/3}$$

$$= \frac{1}{32}$$

$$\therefore \quad \frac{P_2}{P_1} = 32$$

39. $$\eta = 1 - \frac{T_2}{T_1}$$

$$\therefore \quad \eta = 1 - \frac{(27 + 273)}{(273 + 627)}$$

$$= 1 - \frac{300}{900} = \frac{600}{900} = \frac{2}{3}$$

40. Resistance of the lamp

$$R_0 = \frac{V^2}{P}$$

$$= \frac{(30)^2}{90} = 10\,\Omega$$

Current in the lamp

$$i = \frac{30}{10} = 3\Omega$$

When lamp is operated on a 120 V, then resistance

$$R' = \frac{V'}{i} = \frac{120}{3} = 40\,\Omega$$

Thus, for proper glow, the resistance required to the put in series will be

$$R = R' - R_0 = 40 - 10 = 30\,\Omega$$

41. For a two cell battery

$$I = \frac{2E}{2r} = \frac{E}{r}$$

Similarly, for a n cell battery

$$I = \frac{nE}{nr} = \frac{E}{r}$$

So, current in the circuit does not depend on no. of cells in the battery.

Hence, the correct graph will be (c).

42. There are 4 beats between A and B, therefore the possible frequencies of A are 316 or 324 that is (320 ± 4) Hz.

When the prong of A is filed, its frequency becomes greater than the original frequency. If we assume that original frequency of A is 324 then on filing its frequency will be greater then 324. The beats between A and B will be more than 4. But it is given that the beats are again 4, therefore, 324 is not possible.

Therefore, required frequency must be 316 Hz.

(This is true, because on filing the frequency may increase so as to give 4 beats with B of frequency 320 Hz)

43. The frequency of sonometer wire is

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Taking logarithm and differentiate, we get

$$\frac{\Delta n}{n} = \frac{\Delta l}{l} + \frac{1}{2}\frac{\Delta T}{T} + \frac{1}{2}\frac{\Delta m}{m}$$

$$\therefore \quad \frac{\Delta n}{n} = \frac{\Delta l}{l} + 0 + 0$$

$$= 1\%$$

Hence, frequency will increase by 1%.

44. The latent heat of vaporisation of water is large, so when water is sprinkled over a large area, evaporation takes place, thus, causes cooling.

45. The given equation is

$$y = 5 \sin \frac{\pi}{2}(100t - x) \qquad \ldots(\text{i})$$

Comparing Eq. (i) with standard wave equation, given by

$$y = A \sin(\omega t - kx) \quad \ldots(ii)$$

we have

$$\omega = \frac{100\,\pi}{2} = 50\pi$$

$$\Rightarrow \quad \frac{2\pi}{T} = 50\,\pi$$

$$\Rightarrow \quad T = \frac{2\pi}{50\,\pi} = 0.04\ \text{s}$$

46. Loudness of sound note depends on intensity according to relation,

$$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

Here, I_0 is the intensity of minimum audible sound.

Pitch is the characteristics of sound that depends on frequency. It determines the shrillness and graveness of sound.

47. An inertial frame is a non-accelerated frame, so option (b) is correct.

48. If A_0 is the area of orifice at the bottom below the free surface and A that of vessel, time t taken to be empited the tank,

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$$

$$\therefore \quad \frac{t_1}{t_2} = \sqrt{\frac{H_1}{H_2}}$$

$$\Rightarrow \quad \frac{t}{t_2} = \sqrt{\frac{H_1}{H_1/2}}$$

$$\Rightarrow \quad \frac{t}{t_2} = \sqrt{2}$$

$$\therefore \quad t_2 = \frac{t}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$$

$$\approx 7\ \text{min}$$

49. Archimedes' upward thrust will be absent for a fluid, if there were no gravity.

50. In LCR series circuit,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(40)^2 + (60 - 30)^2}$$

$$= \sqrt{1600 + 900} = \sqrt{2500}$$

$$= 50\ \text{V}$$

51. Deuterium and helium are both neutral. Only ions are accelerated by an electric field or magnetic field when they are moving perpendicular to field. Hence, neither of them will be accelerated.

52. Magnetic field at the axis inside the solenoid

$$B = \mu_0 ni$$

Here, $n = 10$ turns/cm $= 1000$ turns/m, $i = 5$ A

$$\therefore \quad B = 4\pi \times 10^{-7} \times 1000 \times 5$$

$$= 2\pi \times 10^{-3}\ \text{T}.$$

53. The Lorentz force acting on the current carrying conductor in the magnetic field is

$$F = IBl \sin\theta$$

Since, wire PQ is parallel to the direction of magnetic field, then $\theta = 0$,

$$\therefore \quad F_{PQ} = IBl \sin 0° = 0$$

Also, wire QR is perpendicular to the direction of magnetic field, then $\theta = 90°$.

$$\therefore \quad F_{QR} = IBl \sin 90° = IBl$$

54. In an AC circuit, the coil of high inductance and negligible resistance used to control current, is called the choke coil. The power factor of such a coil is given by

$$\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\approx \frac{R}{\omega L} \qquad (\text{as } R << \omega L)$$

As $R << \omega L$, $\cos\phi$ is very small. Thus, the power absorbed by the coil is very small. The only loss of energy is due to hysteresis in the iron core, which is much less than the loss of energy in the resistance that can also reduce the current if placed instead of the choke coil.

55. From Kirchhoff's first law at jucntion P,

$$I_1 + I_2 = 6 \quad \ldots(i)$$

From Kirchhoff's second law to the closed circuit $PQRP$,

$$-2I_1 - 2I_1 + 2I_2 = 0$$

$$\Rightarrow \quad -4I_1 + 2I_2 = 0$$

$$\Rightarrow \quad 2I_1 - I_2 = 0 \quad \ldots(ii)$$

Adding Eqs. (i) and (ii), we get

$$3I_1 = 6$$

$$\Rightarrow \quad I_1 = 2\text{A}$$

From Eq. (i),

$$I_2 = 6 - 2 = 4\text{A}$$

56. If trivalent impurity is mixed with pure germanium crystal, the crystal so obtained is called *p*-type semiconductor. Here, aluminium and indium atoms are trivalent impurity atoms.

57. Specific charge of electron,

$$\frac{e}{m} = 1.8 \times 10^{11}\ \text{C kg}^{-1}$$

Maximum kinetic energy of photoelectron

$$\frac{1}{2} m v_{\max}^2 = eV_s$$

where V_s is the stopping potential.

$$\Rightarrow \quad V_s = \frac{m v_{\max}^2}{2e} = \frac{v_{\max}^2}{2(e/m)}$$

$$= \frac{(1.2 \times 10^6)^2}{2 \times 1.8 \times 10^{11}}$$

$$= 0.4 \times 10 = 4\text{V}$$

59. de-Broglie wavelength

$$\lambda = \frac{h}{p}$$

or $$\lambda \propto \frac{1}{p}$$

Hence, the graph between λ and p will be a rectangular hyperbola.

60. $$r = r_0 (A)^{1/3}$$

$$\therefore \quad \frac{r_1}{r_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$= \left(\frac{64}{125}\right)^{1/3}$$

$$= \left[\left(\frac{4}{5}\right)^3\right]^{1/3} = \frac{4}{5}$$

Chemistry

1. Carnallite ($KCl \cdot MgCl_2 \cdot 6H_2O$), dolomite ($MgCO_3 \cdot CaCO_3$) and sea water are the ores of magnesium. Calamine ($ZnCO_3$) is an ore of zinc.

2. Electronic configuration of

$_{28}Ni = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^8, 4s^2$

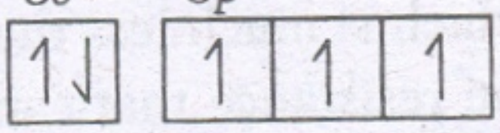
$Ni^{2+} = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^8, 4s^0$

$_{29}Cu = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, 4s^1$

$Cu^+ = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, 4s^0$

So, the given configuration is of Cu^+

3. [Ne] $3s^2$ $3p^3$



Elements having half filled or fully filled orbitals are more stable. Hence, much energy is required to remove an electron from the outermost orbit. So, [Ne] $3s^2 3p^3$ has highest ionisation energy.

4. Bromine has zero oxidation state because it is in free state

$$\overset{0}{Br_2} \longrightarrow \overset{x\ \ -2}{BrO_3^-}$$

$$x + (-2 \times 3) = -1$$

$$x + (-6) = -1$$

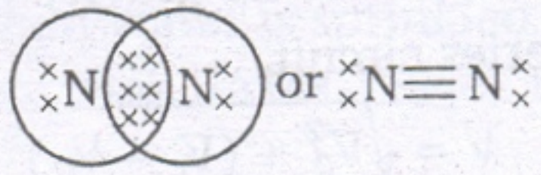
$$x = +6 - 1$$

$$= +5$$

So, oxidation number changes from 0 to +5.

5. Alkali metals have a tendency to lose the single valence electron and form +ve ion and gain inert gas configuration but in case of cesium, the distance of the valence electron is maximum. So, force of attraction by the nucleus is least, hence, it is more reactive.

6. Lewis structure of N_2 molecule is



7. When hydrogen forms hydrogen bonding with fluorine it will be strongest H-bonding because fluorine is strongest electrongative element.

8. Weight of 11.2 dm^3 of CO_2 gas at STP = 44/2
= 22 *g*.

$$\underset{56g}{KOH} + \underset{44g}{CO_2} \longrightarrow KHCO_3$$

KOH required for complete neutralisation requires of 22 g of CO_2 is $= \frac{56}{44} \times 22 = 28$ g

9. We know that

$$PV = nRT$$

or $$PV = \frac{w}{m} RT$$

or $$m = \frac{w}{V} \frac{RT}{P}$$

or $$M = d \frac{RT}{P}$$

$d = 1.964\ g/dm^3 = 1.964 \times 10^{-3}$ g/cc.

$P = 76$ cm $= 1$ atm

$R = 0.0812$ L atm K^{-1} mol^{-1}

$= 82.1$ cc atm K^{-1} mol^{-1}

$T = 273$ K

$$m = \frac{1.964 \times 10^{-3} \times 82.1 \times 273}{1} = 44.$$

The molecular weight of CO_2 is 44.
So, the gas is CO_2.

11.

	$2SO_2$	+	O_2	$\rightleftharpoons$	$2SO_3$
initrally	4		4		0
at equil.	4(1–0.25)		4 (1–0.25)		0.25 × 2

Total number of moles at equilibrium
$= 4(1 - 0.25) + 4(1 - 0.25) + 0.5$
$= 4 - 1 + 4 - 1 + 0.5 = 6.5$

12. Autocatalysis is a process in which one of the product behaves as a catalyst.

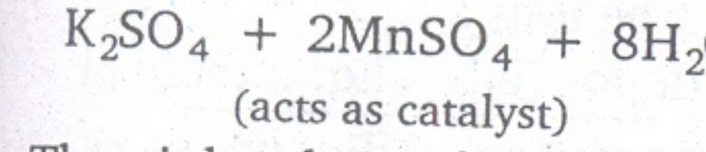
e.g., $2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4 \longrightarrow$
$K_2SO_4 + 2MnSO_4 + 8H_2O + 10CO_2$
(acts as catalyst)

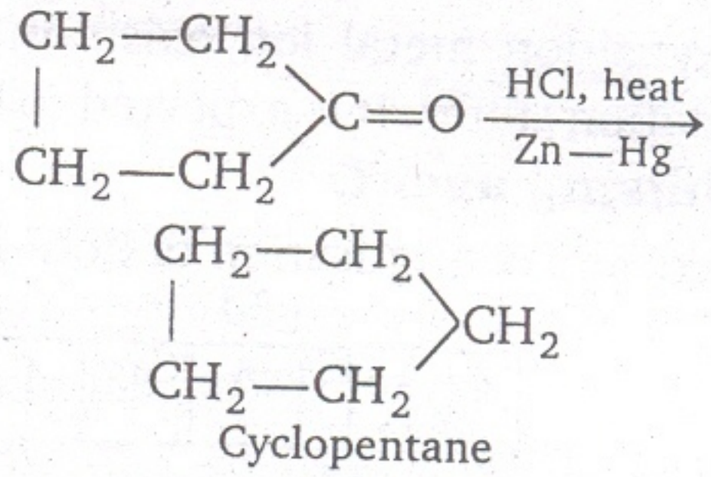
The pink colour of $KMnO_4$ disappears slowly on reaction with oxalic acid, but the rate of disappearance of colour fastens after some time due to the formation of $MnSO_4$ which acts as auto catalyst for the reaction.

13. When organic compound is fused with sodium metal, nitrogen of the compound is converted into sodium cyanide as

$$Na + C + N \longrightarrow NaCN$$

14.



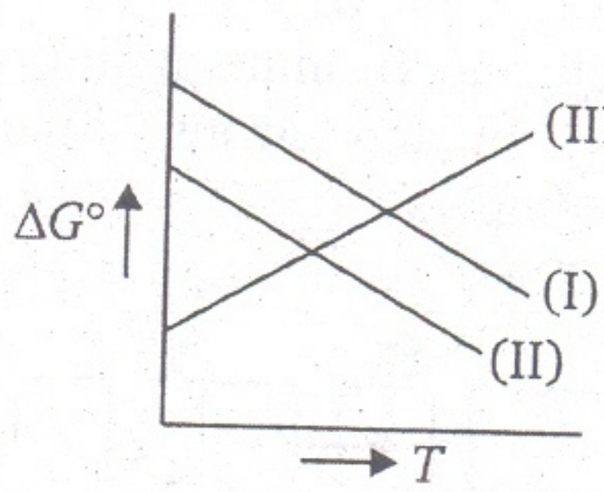


Cyclopentane

15. Williamson's synthesis : It involves the heating of alkyl halide with sodium or potassium alkoxides. This reaction is used for the preparation of ethers *e.g.*,

$$C_2H_5ONa + C_2H_5I \longrightarrow C_2H_5—OC_2H_5 + NaI$$

16.



Reaction (i) $M(s) + \frac{1}{2} O_2 \longrightarrow MO(s)$

Reaction (ii) $CO(g) + \frac{1}{2} O_2 \longrightarrow CO_2(g)$

Reaction (iii) $C(s) + \frac{1}{2} O_2(g) \longrightarrow CO_2(g)$

$\Delta G°$ *vs*, T plot in the Ellingham's diagram slopes downwards for the reaction (I) and reaction (II). For reaction (III) $\Delta G°$ *vs* T in the diagram slopes upwards.

17. Bond breaking processes or decomposition processes are endothermic process.

18. Liquor ammonia bottles are opened only after cooling because it has high vapour pressure and it is mild explosive.

19. The first ionisation energy of xenon is quite close to that of oxygen and the molecular diameter of xenon and oxygen are almost identical.

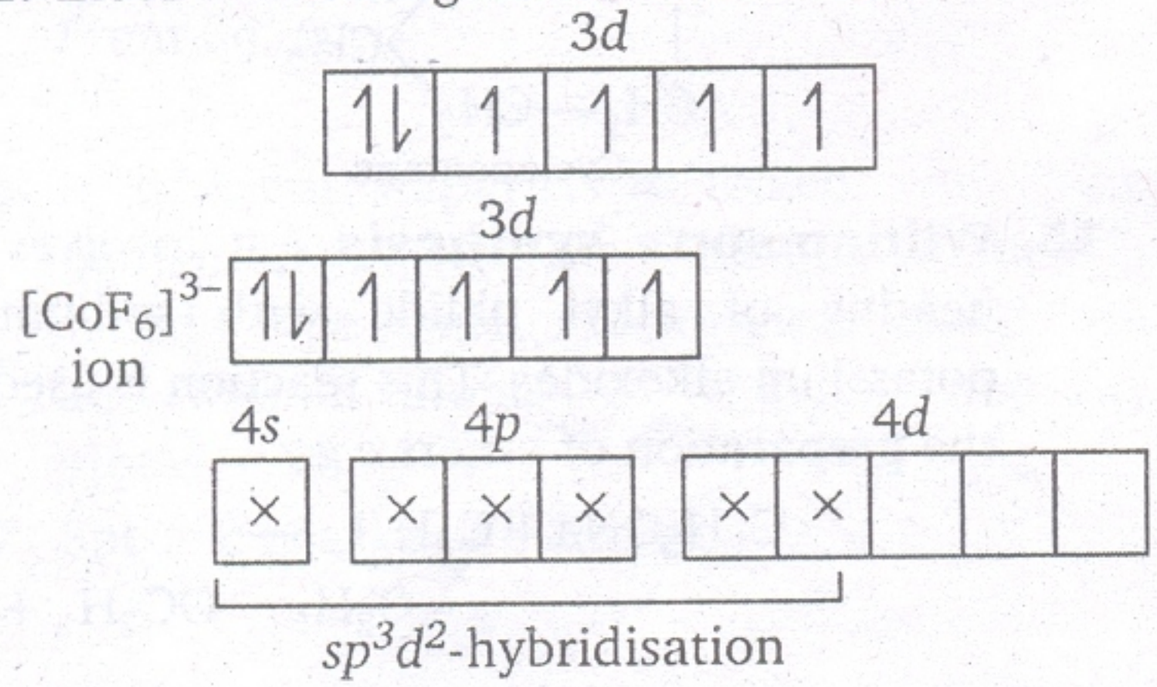
Based on the above facts it is suggested that since oxygen combines with PtF_6, so xenon should also form similar compound with PtF_6.

20. Magnetic moment $= \sqrt{n(n+2)}$ BM

where n is the number of unpaired electrons. Maximum the value of unpaired electron, greater the value of magnetic moment. So, $3d^5$ has highest value of magnetic moment.

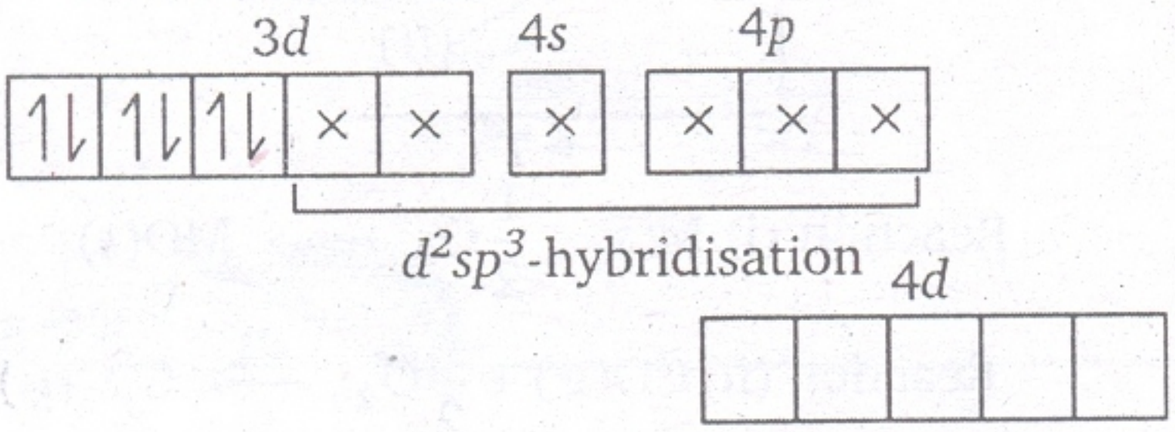
21. A transition metal ion exists in the highest oxidation states. It is expected to behave as an **oxidising agent.**

22. Electronic configuration of Co^{2+} ion

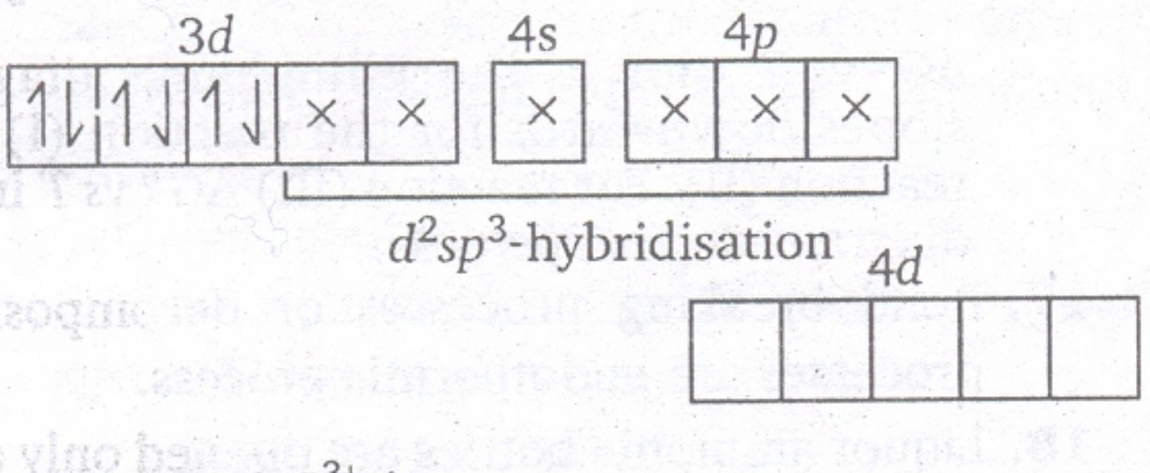


F^- is a weak ligand. It cannot pair up electrons with *d*-subshell and forms outer orbital octahedral complex.

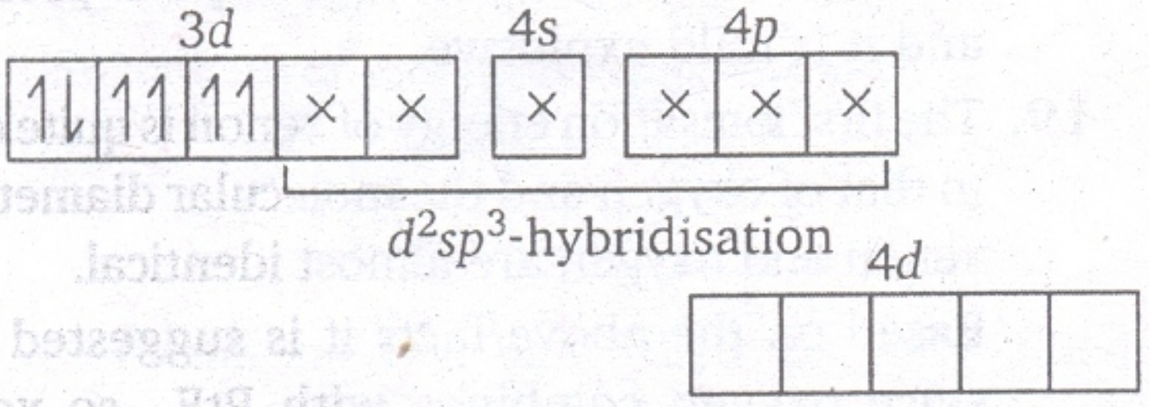
$[Co(NH_3)_6]^{3+}$ ion



$[Fe(CN)_6]^{-3}$ ion



$[Cr(NH_3)_6]^{3+}$ ion



NH_3 and CN^- are strong ligands. So, they form inner orbital complex.

23. NO_2^- can participate in linkage isomerism because it may be bonded to metal through nitrogen or through oxygen.

$$[(NH_3)_5CoNO_2]Cl_2$$
Pentaamminenitro cobalt (III) chloride

and
$$[(NH_3)_5CoONO]Cl_2$$
Pentaamminenitro cobalt (III) chloride

24. Molecular orbital configuration of

$$N_2 = 14e^- = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2, \overset{*}{\sigma} 2s^2,$$
$$\sigma 2p_x{}^2, \pi 2p_y{}^2, \pi 2p_z{}^2$$

$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{10-4}{2} = 3$$

Similarly for O_2 molecule, bond order = 2
For He_2 molecule, bond order = 0
For H_2 molecule bond order = 1
So, N_2 has highest bond order.

25. Molecules or ion having no unpaired electrons are diamagnetic *e.g.*,

$$Li_2 = 6e^- = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2$$

26.
$$k = \frac{2.303}{t} \log \frac{A_0}{A}$$
$$= \frac{2.303}{40} \log \frac{0.1}{0.005}$$
$$= \frac{2.303}{40} \log 20 = 0.075$$

Rate of reaction when concentration of *X* is 0.01 M will be
$0.075 \times 0.01 = 7.5 \times 10^{-4}$ M min^{-1}

27. Reactions having low E_a are fast reactions and reactions having high E_a are slow reactions.
If the E_a is high, then the number of effective collisions will be small and the reaction will be slow.

28. Solid NaCl does not conduct electricity due to absence of free ions.

29. $$\frac{\text{Wt. of Cu deposited}}{\text{Wt. of } H_2 \text{ produced}} = \frac{\text{Eq. wt. of Cu}}{\text{Eq. wt. of H}}$$

$$\frac{0.16}{\text{wt. of } H_2} = \frac{64/2}{1} = \frac{32}{1}$$

$$\text{wt. of } H_2 = \frac{0.16}{32} = 5 \times 10^{-3} \text{ g}$$

Volume of H_2 liberated at STP
$$= \frac{22400}{2} \times 5 \times 10^{-3} \text{ cc}$$
$$= 56 \text{ cc}$$

30. A salt is precipitated only when the product of ionic concentration is more then its solubility product.

$$K_{sp} = 1 \times 10^{-8}$$

$$[A^+] = 10^{-3}\ \text{M}$$

$$[B^-] = \frac{1 \times 10^{-8}}{10^{-3}} = 10^{-5}\ \text{M}$$

So, AB will be precipitated only when the concentration of $[B^-]$ is more than 10^{-5} M.

31. Since reduction occurs at Ag electrode hence, increase in the concentration of Ag^+ or decrease in concentration of Cu^{2+} will increase the voltage.

32. From H_2O, $[H^+] = 1 \times 10^{-7}$ M

From HCl, $[H^+] = 1 \times 10^{-8}$ M

$$\text{Total}\,[H^+] = (1 \times 10^{-7} + 1 \times 10^{-8})\ \text{M}$$

$$= (1 \times 10^{-7} + 0.1 \times 10^{-7})\ \text{M}$$

$$= 1.1 \times 10^{-7}\ \text{M}$$

$$\text{pH} = -\log (1.1 \times 10^{-7}) = 6.9586$$

33. $\dfrac{P - P_s}{P} = \dfrac{w_1 M_2}{w_2 M_1}$

To produce same lowering of vapour pressure, $\dfrac{P - P_s}{P}$ will be same for both cases.

So,

$$\frac{W_{(\text{Glucose})} \times 18}{50 \times 180} = \frac{W_{(\text{urea})} \times 18}{50 \times 60}$$

$W_{(\text{Glucose})}$ = weight of glucose

$W_{(\text{Glucose})}$ = weight of urea

or $$\frac{W_{(\text{Glucose})} \times 18}{50 \times 180} = \frac{1 \times 18}{50 \times 60}$$

$$W_{(\text{Glucose})} = 3$$

34. Benzoic acid in benzene exist as a dimer. So, number of molecules decreases and hence, osmotic pressure decreases.

35. $\Delta G = \Delta H - T\Delta S$

For a reaction to be spontaneous ΔG must be negative. If ΔG and ΔH are negative, then reaction will be spontaneous at all temperature.

36. Flocculation value $\propto \dfrac{1}{\text{Coagulating power}}$

$Fe(OH)_3$ is a positively charged sol.

To coagulate $Fe(OH)_3$, –ve charge electrolyte is used and greater the value of –ve charge, coagulating power will be strong. Among the given electrolytes, NaCl has lowest coagulating power. So, its flocculation value will be maximum.

37. $X(g) + 3Y(g) \rightleftharpoons 2Z(g)$

$$\Delta S^\circ = 2S^\circ(Z) - \{S^\circ(X) + 3S^\circ(Y)\}$$

$$= 2 \times 50 - \{60 + 3 \times 40\}$$

$$= 100 - 180 = -80\ \text{J K}^{-1}\ \text{mol}^{-1}$$

Given $\Delta H^\circ = -40$ kJ $= -40{,}000$ J

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$

At equilibrium, $\Delta G^\circ = 0$

$\therefore$ $$\Delta H^\circ = T\Delta S^\circ$$

or $$T = \frac{\Delta H^\circ}{\Delta S^\circ} = \frac{40{,}000}{80} = 500\ \text{K}$$

38. NaCl has fcc structure.

In fcc lattice

$$r^+ + r^- = \frac{a}{2}$$

where a = edge length

$$r^+ = 95\ \text{pm} \quad r^- = 181\ \text{pm}$$

Edge length $= 2r^+ + 2r^-$

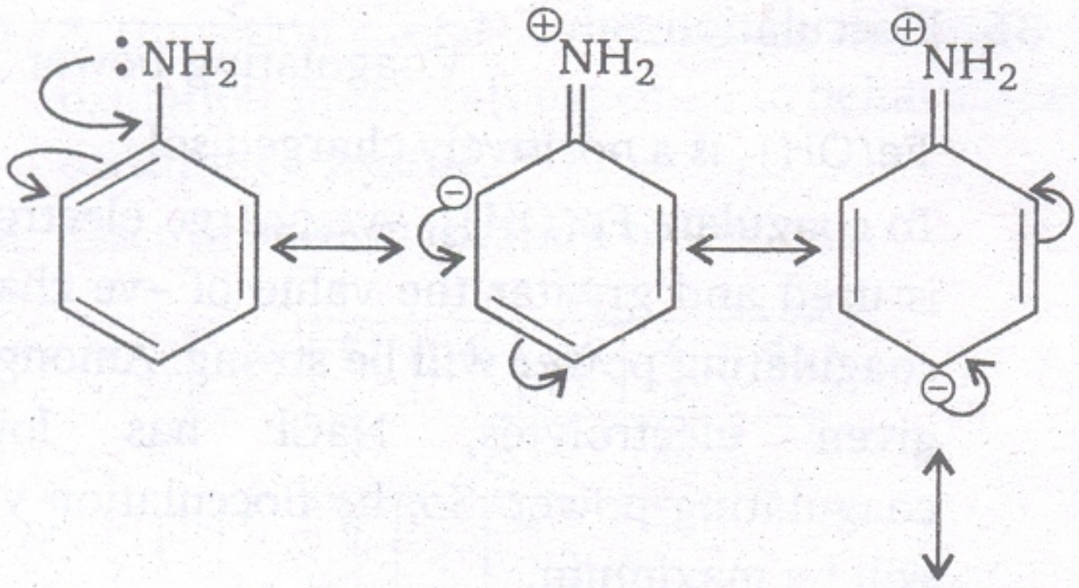
$$= (2 \times 95 + 2 \times 181)\ \text{pm}$$

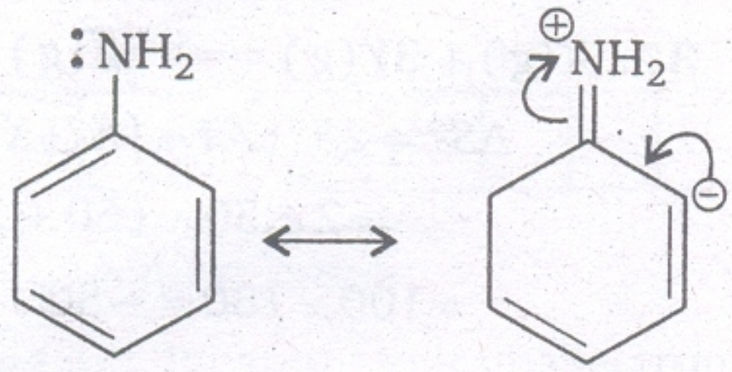
$$= 190 + 362 = 552\ \text{pm}$$

39. During inductive effect shifting of σ electrons takes place due to which partially charges are developed on the atom

$$\overset{+\delta''}{\text{C}} - \overset{+\delta'}{\text{C}} - \overset{+\delta}{\text{C}} - \overset{-\delta}{\text{Cl}}$$

40. $-NH_2$ has $+R$ effect, it donates electrons to the benzene ring. As a result, the lone pair of electron on the N-atom gets delocalized over the benzene ring and thus it is less readily available for protonation. Hence, aniline is a weaker base than cyclohexylamine.



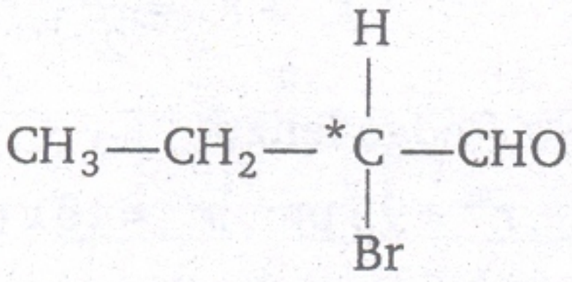


Resonance structure of aniline.

41. Wurtz Reaction : Alkyl halide reacts with sodium in presence of dry ether forms alkane is known as Wurtz reaction.

$$CH_3Br + 2Na + BrCH_3 \xrightarrow{\text{ether}} CH_3 — CH_3 + 2NaBr$$

42. Compounds having asymmetric C-atom is optically active *e.g.*,



The C-atom whose four valencies are satisfied by four different groups is asymmetric C-atom.

43. In cyclopropane the angle strain is maximum. Hence, it is highly strained molecule and consequently most unstable. The angle strain in cyclobutane is less than cyclopropane. Hence, cyclobutane is more stable. This stability increases upto 6 membered rings, then decreases from 7 to 11 membered rings and from the 12 membered rings onwards attains the stability of 6 membered ring. Heat of combustion is a method of measuring chemical stability. Hence, cyclohexane has the lowest heat of combustion.

44. —OH group is converted into —Cl group by $SOCl_2$ or anhy. $ZnCl_2$/conc. HCl or HCl etc.

45. $R — X \xrightarrow[KCN]{\text{Alcoholic}} R — CN$

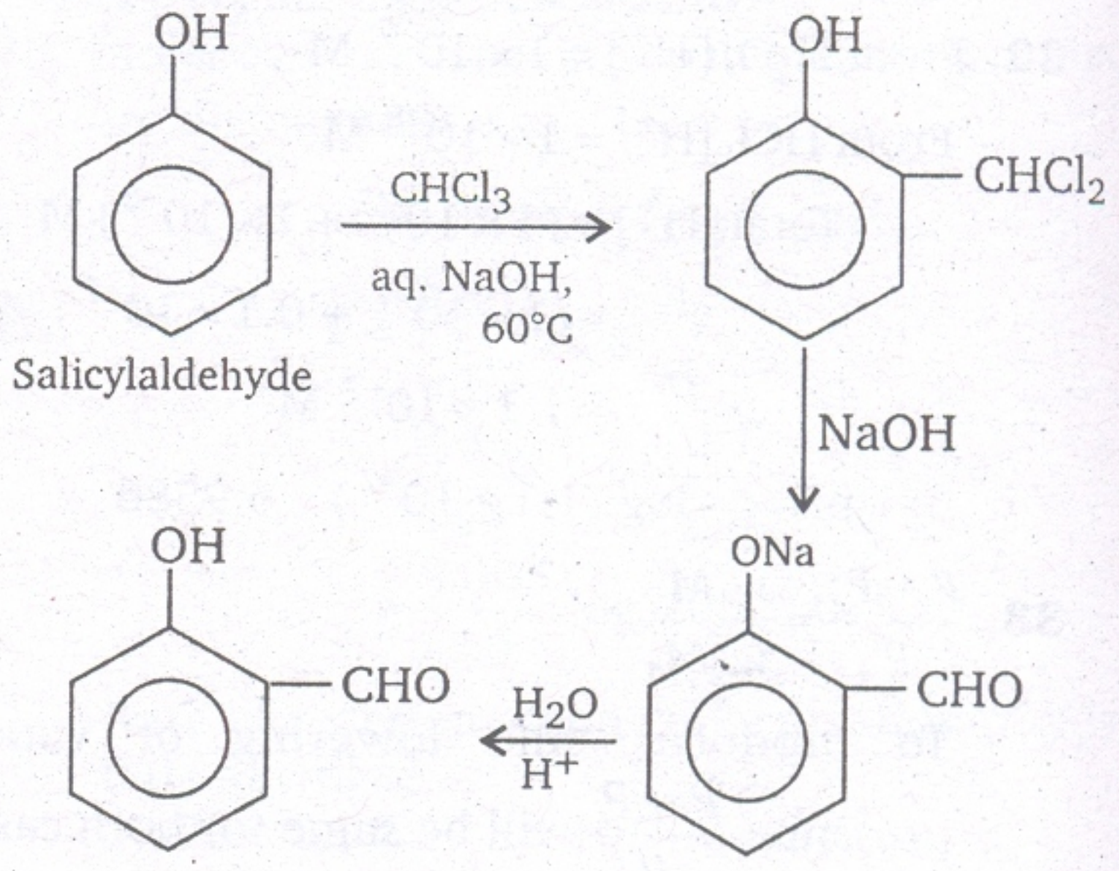
Alkyl cyanide

$\xrightarrow{\text{dil. HCl}} R — COOH$

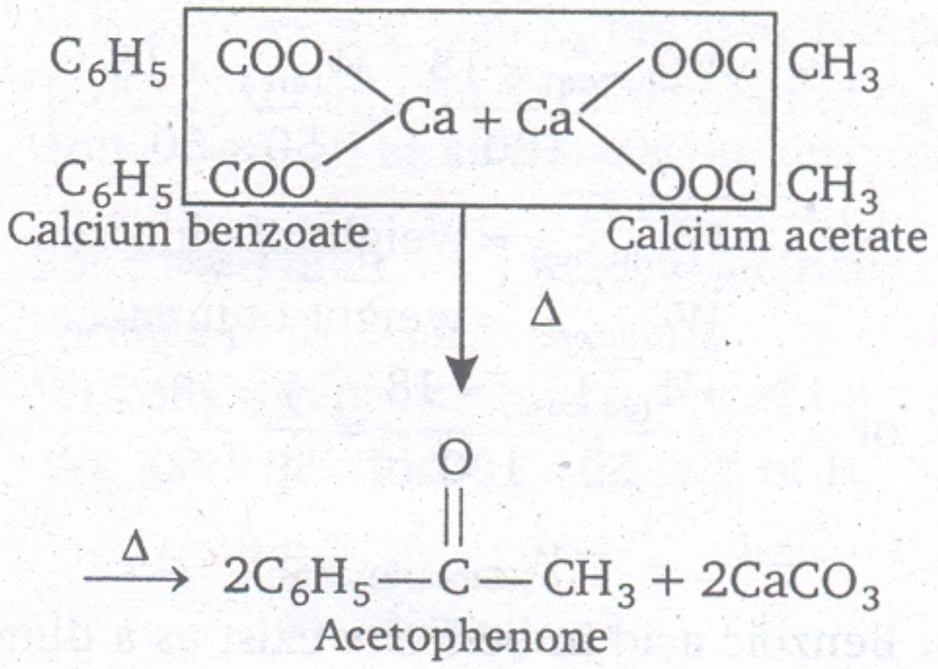
Carboxylic acid

46. Phenol does not decompose sodium carbonate or sodium bicarbonate *i.e.*, CO_2 is not evolved, because phenol is a weaker acid than carbonic acid.

47. Reimer-Tiemann Reaction : In this reaction phenol reacts with chloroform and alkali to form salicylaldehyde.



48.



49. Benzyl chloride on hydrolysis gives benzyl alcohol.

$$\underset{\text{Benzyl chloride}}{C_6H_5CH_2Cl} \xrightarrow[\text{boil}]{\text{aq. NaOH}} \underset{\text{Benzyl alcohol}}{C_6H_5CH_2OH} + NaCl$$

50. Hofmann Reaction : In this reaction acid amide group reacts with Br_2 in presence of NaOH or KOH to give primary amine group.

The amine is one carbon less than the parent amide. So, the reaction is known as Hofmann degradation reaction.

$$\underset{\text{Acid amide}}{R\text{—}CONH_2} + Br_2 + 4KOH \xrightarrow{\Delta}$$

$$\underset{\text{Primary amine}}{RNH_2} + 2KBr + K_2CO_3 + 2H_2O$$

51. Structure of glucose is

CHO } Aldehyde group
CHOH
CHOH
CHOH
CHOH
CH_2OH} *Pri.* alcoholic group

Sec. alcoholic group

52. Fats are also known as triglycerides. These triglycerides are the triesters of faty acid with glycerol. So, the characteristic feature of fat is **ester group.**

53. At pH = 4, an amphoteric Zwitter ion structure changes into cation when an acid is added to it.

$$\underset{\oplus NH_3}{R\text{—}CH\text{—}COO^-} \xrightarrow{+H^+} \underset{\oplus NH_3}{R\text{—}CH\text{—}COOH}$$

54. Insulin regulates the metabolism of carbohydrates (glucose).

55. Mohr's salt is $(NH_4)_2SO_4 \cdot FeSO_4 \cdot 6H_2O$

The equation is :

$$5Fe^{2+} + MnO_4^- + 8H^+ \longrightarrow 5Fe^{3+} + Mn^{2+} + 4H_2O$$

Total change in oxidation number of iron

$$= (+3) - (+2)$$
$$= +1$$

So, equivalent wt. of Mohr's salt

$$= \frac{\text{Mol. wt. of Mohr's salt}}{1}$$
$$= \frac{392}{1}$$
$$= 392$$

56. The brown ring is obtained due to the formation of pentaaquanitrosonium ion. In this reaction reduction of nitrate ion by Fe^{2+} ion takes places as

$$NO_3^- + 3Fe^{2+} + 4H^+ \longrightarrow NO + 3Fe^{3+} + 2H_2O$$

$$Fe^{2+} + NO + 5H_2O \longrightarrow \underset{\text{Brown complex}}{[Fe(H_2O)_5NO]^{2+}}$$

57. When fat is heated in presence of $KHSO_4$ (dehydrating agent) the glycerol portion of the molecule is dehydrated and form unsaturated aldehyde $CH_2{=}CH - CHO$ (acrolein), a bed smelling compound. It is the test for fat.

58. Halogen containing compounds (C_6H_5Cl)
When placed in a flame, the presence of halogen is revealed by a green to blue flame.

59. $A + B \longrightarrow C + D$

Rate $(r) = k[A]^a [B]^b$ ...(i)

$2r = k[2A]^a [B]^b$ ...(ii)

$3r = k[A]^a [9B]^b$ ...(iii)

Dividing Eq. (ii) by Eq. (i)

$$2 = 2^a \text{ or } a = 1$$

Dividing Eq. (iii) by Eq. (i)

$$3 = 9^b \text{ or } 3 = 3^{2b} \text{ or } 2b = 1 \text{ or } b = \frac{1}{2}$$

So, order of reaction $= 1 + \frac{1}{2} = \frac{3}{2}$

$$\Delta T_b = i \cdot K_b \cdot M$$

60. Molecules having maximum number of ions will have maximum value of boiling point. In the given options, 0.01 M Na_2SO_4 contains maximum number of ions. So, 0.01 M Na_2SO_4 has highest boiling point.

1. Truth Table

p	q	r	$\sim p$	$\sim r$	$p \wedge \sim r$	$\sim p \vee (p \wedge \sim r)$	$(\sim p \vee q)$	$(p \wedge \sim r) \rightarrow (\sim p \vee q)$
T	T	T	F	F	F	T	T	
T	T	F	F	T	T	T	T	
T	F	T	F	F	F	F	F	
T	F	F	F	T	T	F	F	
F	T	T	T	F	F	T	T	
F	T	F	T	T	F	T	T	
F	F	T	T	F	F	T	T	
F	F	F	T	T	F	T	T	

Since $(p \wedge \sim r) \rightarrow (\sim p \vee q)$ is F

Then, $p = T,\ q = F,\ r = F$

2. Since α, β and γ are the roots of the equation $x^3 - 8x + 8 = 0$, then

$$\alpha + \beta + \gamma = 0,\ \alpha\beta + \beta\gamma + \gamma\alpha = -8,\ \alpha\beta\gamma = -8 \quad \ldots(i)$$

$$\therefore \quad (\alpha + \beta + \gamma)^2 = 0$$

$$\Rightarrow \quad \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = 0$$

$$\Rightarrow \quad \Sigma\alpha^2 = -2(-8) \qquad \text{(From (i))}$$

$$= 16$$

and $$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\gamma + \alpha + \beta}{\alpha\beta\gamma}$$

$$\Rightarrow \quad \frac{1}{\Sigma\alpha\beta} = \frac{0}{-8} = 0 \qquad \text{(From (i))}$$

3. Now, $1080 \div 675$

$$675)1080(1$$
$$\underline{675}$$
$$405)675(1$$
$$\underline{405}$$
$$270)405(1$$
$$\underline{270}$$
$$135)270(2$$
$$\underline{270}$$
$$0$$

$\therefore$ The gcd of 1080 and 675 is 135.

4. Since $a \mid (b + c)$ and $a \mid (b - c)$

$$\Rightarrow \quad \frac{b + c}{a} \text{ and } \frac{b - c}{a}$$

$$\therefore \quad \frac{b + c}{a} \cdot \frac{b - c}{a} = \frac{b^2 - c^2}{a^2}$$

$$\Rightarrow \quad a^2 \mid (b^2 - c^2)$$

$$\Rightarrow \quad b^2 \equiv c^2 \pmod{a^2}$$

5. By taking option (c)

$$a \mid b(+c) \not\Rightarrow a \mid b \text{ and } a \mid c$$

e.g., $\quad 6 \mid 18 \not\Rightarrow 6 \mid 1$ and $6 \mid 17$

6. We have

$$x = 4(1 + \cos\theta) \text{ and } y = 3(1 + \sin\theta)$$

$$\Rightarrow \quad \cos\theta = \frac{x}{4} - 1 \text{ and } \sin\theta = \frac{y}{3} - 1$$

We know

$$\cos^2\theta + \sin^2\theta = 1$$

$$\Rightarrow \quad \left(\frac{x}{4} - 1\right)^2 + \left(\frac{y}{3} - 1\right)^2 = 1$$

$$\Rightarrow \quad \frac{(x - 4)^2}{16} + \frac{(y - 3)^2}{9} = 1$$

7. Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Distance between foci $= 2ae$

and distance between directrices $= \frac{2a}{e}$

According to the question

$$\frac{2ae}{2a/e} = \frac{3}{2}$$

$$\Rightarrow \quad e^2 = \frac{3}{2}$$

We know,

$$b^2 = a^2(e^2 - 1)$$

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{3}{2} - 1 = \frac{1}{2}$$

$$\Rightarrow \quad \frac{b}{a} = \frac{1}{\sqrt{2}}$$

$\therefore$ Required ratio $a : b$ is $\sqrt{2} : 1$.

8. Equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1,\ a > b$ and equation of hyperbola is $\frac{x^2}{25} - \frac{y^2}{16} = 1,\ a > b$.

Let e and e' be the eccentricities of the ellipse and hyperbola.

$$\therefore \quad e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{\frac{25 - 16}{25}}$$

$$= \frac{3}{5}$$

$$\text{and} \quad e' = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{\frac{25 + 16}{25}}$$

$$= \frac{\sqrt{41}}{5}$$

(i) Centre of ellipse is (0, 0) and centre of hyperbola is (0, 0).

(ii) Foci of ellipse are $(\pm ae, 0)$ or $(\pm 15, 0)$ foci of hyperbola are $(\pm ae', 0)$ or $(\pm \sqrt{41}, 0)$.

(iii) Directrices of ellipse are $x = \pm \frac{a}{e}$

$\Rightarrow x = \pm \frac{25}{3}$ directrices of hyperbola are

$$x = \pm \frac{a}{e}$$

$$\Rightarrow \quad x = \pm \frac{25}{\sqrt{41}}$$

(iv) Vertices of ellipse are $(\pm a, 0)$ or $(\pm 5, 0)$
Vertices of hyperbola are $(\pm a, 0)$ or $(\pm 5, 0)$

From the above discussions, their are common in centre and vertices.

9. Given that $\sec\theta = m$ and $\tan\theta = n$

$$\therefore \quad \frac{1}{m}\left[(m+n) + \frac{1}{(m+n)}\right]$$

$$= \frac{1}{\sec\theta}\left[\sec\theta + \tan\theta + \frac{1}{\sec\theta + \tan\theta}\right]$$

$$= \frac{[\sec^2\theta + \tan^2\theta + 2\sec\theta\tan\theta + 1]}{\sec\theta(\sec\theta + \tan\theta)}$$

$$= \frac{2\sec^2\theta + 2\sec\theta\tan\theta}{\sec\theta(\sec\theta + \tan\theta)}$$

$$= \frac{2\sec\theta(\sec\theta + \tan\theta)}{\sec\theta(\sec\theta + \tan\theta)}$$

$$= 2$$

10. $\dfrac{\sin 85° - \sin 25°}{\cos 65°}$

$$= \frac{2\cos\frac{85° + 35°}{2}\sin\frac{85° - 35°}{2}}{\cos 65°}$$

$$= \frac{2\cos 60°\sin 25°}{\cos(90° - 25°)} = \frac{2\cdot\frac{1}{2}\cdot\sin 25°}{\sin 25°}$$

$$= 1$$

11. Let point $P(x_1, y_1)$ be any point on the circle, therefore it satisfy the circle

$$(x_1 - 3)^2 + (y_1 + 2)^2 = 5r^2 \quad \ldots(i)$$

The length of the tangent drawn from point $P(x_1, y_1)$ to the circle $(x-3)^2 + (y+2)^2 = r^2$ is

$$\sqrt{(x_1 - 3)^2 + (y_1 + 2)^2 - r^2}$$

$$= \sqrt{5r^2 - r^2} \qquad \text{(From (i))}$$

$$\Rightarrow \quad 16 = 2r$$

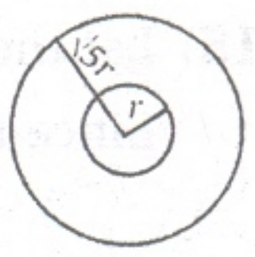
$$\Rightarrow \quad r = 8$$

$\therefore$ The area between two circles

$$= \pi 5r^2 - \pi r^2$$

$$= 4\pi r^2 = 4\pi \times 8^2$$

$$= 256\pi \text{ sq unit}$$



12. Given equation of circles are

$$ax^2 + ay^2 + 2g_1x + 2f_1y + c_1 = 0$$

and $bx^2 + by^2 + 2g_2x + 2f_2y + c_2 = 0$

Or it can be rewritten as

$$x^2 + y^2 + \frac{2g_1}{a}x + \frac{2f_1}{a}y + \frac{c_1}{a} = 0$$

and $x^2 + y^2 + \frac{2g_2}{b}x + \frac{2f_2}{b}y + \frac{c_2}{b} = 0$

$\therefore$ Centres of circles are $\left(-\frac{g_1}{a}, -\frac{f_1}{a}\right)$ and $\left(-\frac{g_2}{b}, -\frac{f_2}{b}\right)$ respectively.

We know, if two circles cut orthogonally, then

$$2(g_1g_2 + f_1f_2) = c_1 + c_2$$

$$\Rightarrow \quad 2\left(\frac{g_1g_2}{ab} + \frac{f_1f_2}{ab}\right) = \frac{c_1}{a} + \frac{c_2}{b}$$

$$\Rightarrow \quad 2(g_1g_2 + f_1f_2) = bc_1 + ac_2$$

13. Let $S_1 \equiv x^2 + y^2 - 6x - 12y + 37 = 0$

and $S_2 \equiv x^2 + y^2 - 6y + 7 = 0$

The equation of common tangent of the two circles is $S_1 - S_2 = 0$

$$\Rightarrow \quad x^2 + y^2 - 6x - 12y + 37 - (x^2 + y^2 - 6y + 7) = 0$$

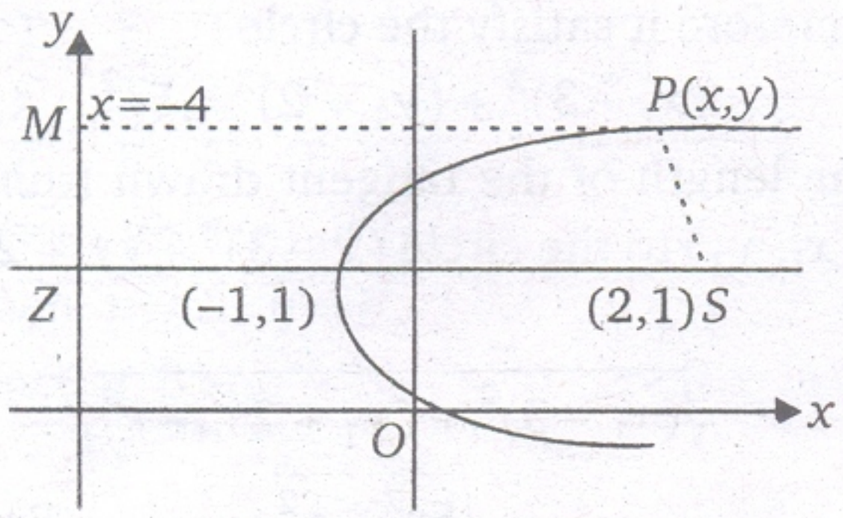
$$\Rightarrow \quad -6x + 6y + 30 = 0$$

$$\Rightarrow \quad x - y - 5 = 0$$

14. By the condition of parabola

$$PM^2 = PS^2$$

$$\Rightarrow \quad (x+4)^2 = (x-2)^2 + (y-1)^2$$



$$\Rightarrow x^2 + 8x + 16 = x^2 - 4x + 4 + y^2 - 2y + 1$$

$$\Rightarrow \quad y^2 - 2y - 12x - 11 = 0$$

15. Let the equation of line be $y = mx + c$.

Since this line is tangent to the circle

$$x^2 + y^2 = 5$$

$$\therefore \quad c = \pm a\sqrt{1+m^2}$$

$$= \pm\sqrt{5}\sqrt{1+m^2} \qquad \ldots(i)$$

Also, the above line is tangent to the parabola

$$y^2 = 40x$$

$$\therefore \quad c = \frac{a}{m} = \frac{10}{m} \qquad \ldots(ii)$$

From Eqs. (i) and (ii)

$$\Rightarrow \quad \frac{10}{m} = \pm\sqrt{5}\sqrt{1+m^2}$$

$$\Rightarrow \quad \frac{20}{m^2} = 1 + m^2$$

$$\Rightarrow \quad m^4 + m^2 - 20 = 0$$

$$\Rightarrow \quad (m^2+5)(m^2-4) = 0$$

$$\Rightarrow \quad m^2 = 4,\ m^2 \neq -5$$

$$\Rightarrow \quad m = \pm 2$$

$$\Rightarrow \quad c = \pm 5$$

$$\therefore \quad y = \pm 2x \pm 5$$

16. We have

$$2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} \qquad \ldots(i)$$

and $$A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix} \qquad \ldots(ii)$$

Multiply Eq. (ii) by 2 and subtracting Eq.(i) from (ii), we get

$$B = 2\begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$$

17. Given that $O(A) = 2\times 3$, $O(B) = 3\times 2$ and $O(C) = 3\times 3$

$$\Rightarrow \quad O(A') = 3\times 2, O(B') = 2\times 3$$

(a) $CB + A'$

Now order of CB = (order of C) (order of B)

= (order of C is 3×3) (order of B is 3×2)

= order of CB is 3×2

Since $O(A') = 3\times 2$

$\therefore$ Matrix $CB + A'$ can be determined.

(b) $O(BA) = 3\times 3$

and $O(C) = 3\times 3$

$\therefore$ Matrix BAC can be determined.

(c) $C(A + B')'$

$O(A + B') = 2\times 3$

$\Rightarrow \quad O(A + B')' = 3\times 2$

and $O(C) = 3\times 3$

$\therefore$ Matrix $C(A + B')'$ can be determined.

(d) $C(A + B')$

$O(A + B') = 2\times 3$

and $O(C) = 3\times 3$

$\therefore$ Matrix $C(A + B')$ cannot be determined

$\therefore$ Option (d) is correct.

18. $A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$

$$\therefore \quad A^2 - 4A + 10I = A$$

$$\Rightarrow \quad \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} - 4\begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} + 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5 & -3-3k \\ 2+2k & -6+k^2 \end{bmatrix} - \begin{bmatrix} 4 & -12 \\ 8 & 4k \end{bmatrix} + \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 9-3k \\ -6+2k & 4+k^2-4k \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$\Rightarrow$ $9-3k=-3, -6+2k=2$ ...(i)

and $4+k^2-4k=k$

$\Rightarrow$ $k^2-5k+4=0$

$\Rightarrow$ $(k-4)(k-1)=0$

$\Rightarrow$ $k=4, 1$

But $k=1$ is not satisfied the Eq. (i).

19. Let $A = \begin{vmatrix} x+y & y+z & z+x \\ x & y & z \\ x-y & y-z & z-x \end{vmatrix}$

Applying

$$C_1 \to C_1 + C_2 + C_3$$

$$= \begin{vmatrix} 2(x+y+z) & y+z & z+x \\ x+y+z & y & z \\ 0 & y-z & z-x \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 2 & y+z & z+x \\ 1 & y & z \\ 0 & y-z & z-x \end{vmatrix}$$

Applying $R_2 \to 2R_2 - R_1$

$$= (x+y+z)\begin{vmatrix} 2 & y+z & z+x \\ 0 & y-z & z-x \\ 0 & y-z & z-x \end{vmatrix}$$

$= 0$ ($\because$ Two rows are identical)

20. Let us assume $a * b = a + b + ab$

Now, $a*(b*c) = a*(b+c+bc)$

$= a+b+c+ab+ac+abc$

$= a+b+ab+c+ac+abc$

$= (a*b)*c$

$\therefore$ It is associative

Now $a*b = a+b+ab$

$= b+a+ba$

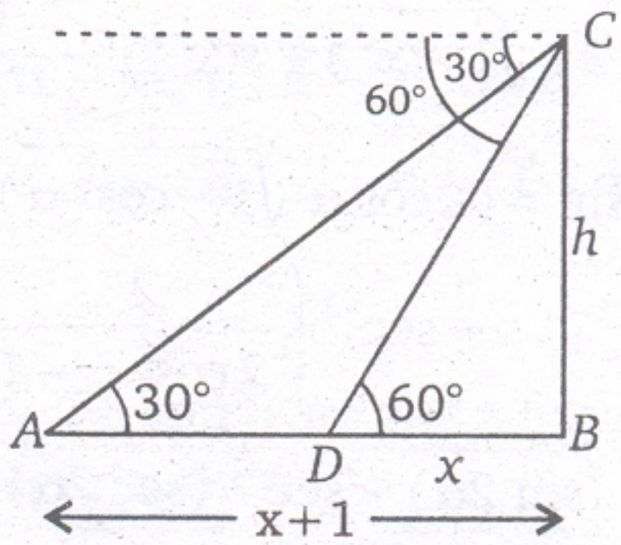
$= b*a$

Also, it is commutative.

$\therefore$ Our assuming operation is true.

21. Let the distance of two consecutive stones are $x, x+1$.

In ΔBCD,



$$\tan 60° = \frac{h}{x}$$

$\Rightarrow$ $x = \frac{h}{\sqrt{3}}$ ...(i)

In ΔABC,

$$\tan 30° = \frac{h}{x+1}$$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{h}{x+1}$

$\Rightarrow$ $\frac{h}{\sqrt{3}} + 1 = \sqrt{3}\,h$ (From (i))

$\Rightarrow$ $\frac{2h}{\sqrt{3}} = 1$

$\Rightarrow$ $h = \frac{\sqrt{3}}{2}$ km

22. Let the angles of a triangle are $3\theta, 4\theta, 5\theta$.

We know

$$\angle A + \angle B + \angle C = 180°$$

$\Rightarrow$ $3\theta + 4\theta + 5\theta = 180°$

$\Rightarrow$ $12\theta = 180°$

$\Rightarrow$ $\theta = 15°$

$\therefore$ Angles are 45°, 60°, 75°

Now, $\sin A = \sin 45° = \frac{1}{\sqrt{2}}$

$\sin B = \sin 60° = \frac{\sqrt{3}}{2}$

$\sin C = \sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore$ $a : b : c = \sin A : \sin B : \sin C$

$= \frac{1}{\sqrt{2}} : \frac{\sqrt{3}}{2} : \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= 2 : \sqrt{6} : \sqrt{3}+1$

23. Given that $\cos^{-1} x = \alpha,\ 0 < x < 1$ ...(i)

$\Rightarrow \quad x = \cos\alpha$

$\therefore \sin^{-1}(2x\sqrt{1-x^2}) + \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \frac{2\pi}{3}$

$\Rightarrow \quad \sin^{-1}(2\cos\alpha\sqrt{1-\cos^2\alpha}) + \sec^{-1}\left(\frac{1}{2\cos^2\alpha - 1}\right) = \frac{2\pi}{3}$

$\Rightarrow \quad \sin^{-1}(\sin 2\alpha) + \sec^{-1}(\sec 2\alpha) = \frac{2\pi}{3}$

$\Rightarrow \quad 2\alpha + 2\alpha = \frac{2\pi}{3}$

$\Rightarrow \quad \alpha = \frac{\pi}{6}$ From (i)

Now, $x = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad 2x = \sqrt{3}$

$\therefore \quad \tan^{-1}(2x) = \tan^{-1}(\sqrt{3})$

$= \frac{\pi}{3}$

24. $\tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}\left(\frac{a+b}{a-b}\right)$

$= \tan^{-1}\left(\frac{\frac{a}{b} + \frac{a+b}{a-b}}{1 - \frac{a}{b}\left(\frac{a+b}{a-b}\right)}\right)$

$= \tan^{-1}\left(\frac{a^2 - ab + ab + b^2}{ab - b^2 - a^2 - ab}\right)$

$= \tan^{-1}\left(-\frac{a^2+b^2}{a^2+b^2}\right) = \tan^{-1}(-1)$

$\therefore$ The value is neither depends on a nor b.

25. (a) $\operatorname{cosec}\theta - \sec\theta = \operatorname{cosec}\theta \cdot \sec\theta$

$\Rightarrow \quad \frac{\cos\theta - \sin\theta}{\cos\theta\sin\theta} = \frac{1}{\cos\theta\sin\theta}$

$\Rightarrow \quad \cos\theta = 1 + \sin\theta$

$\therefore$ At $\theta = 0$ above equation satisfies.

(b) $\operatorname{cosec}\theta \cdot \sec\theta = 1$

$\Rightarrow \quad \sin\theta\cos\theta = 1$

$\Rightarrow \quad 2\sin\theta\cos\theta = 2$

$\Rightarrow \quad \sin 2\theta = 2$

As we know $\sin\theta$ is not greater than 1.

$\therefore$ The above equation has no solution exist.

26. Given that

$A = \{a, b, c\}, B = \{b, c, d\}$ and $C = \{a, d, c\}$

Now, $A - B = \{a, b, c\} - \{b, c, d\}$

$= \{a\}$

and $B \cap C = \{b, c, d\} \cap \{a, d, c\}$

$= \{c, d\}$

$\therefore \quad (A - B) \times (B \cap C) = \{a\} \times \{c, d\}$

$= \{(a, c), (a, d)\}$

27. Since $f : X \to Y$, then $f(x) = \sin x$

Now, take option (c).

Domain $= \left[0, \frac{\pi}{2}\right]$, Range $= [-1, 1]$

For every value of x, we get unique value of y. But the value of y in $[-1, 0)$ does not have any pre-image.

$\therefore$ Function is one-one but not onto.

28. We have,

$\log_4 2 + \log_4 4 + \log_4 x + \log_4 16 = 6$

$\Rightarrow \quad \log_4(2 \times 4 \times x \times 16) = 6$

$\Rightarrow \quad 128x = 4^6$

$\Rightarrow \quad 4^3 \times 2x = 4^6$

$\Rightarrow \quad x = \frac{4^3}{2}$

$\Rightarrow \quad x = 32$

29. Given that

$S_n = \frac{1}{6\cdot 11} + \frac{1}{11\cdot 16} + \frac{1}{16\cdot 21} + \ldots + n \text{ terms}$

$= \frac{1}{5}\left(\frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \ldots + \frac{1}{5n+1} - \frac{1}{5n+6}\right)$

$= \frac{1}{5}\left(\frac{1}{6} - \frac{1}{5n+6}\right)$

$= \frac{1}{5}\left(\frac{5n+6-6}{6(5n+6)}\right)$

$= \frac{n}{6(5n+6)}$

$\Rightarrow \quad 6S_n = \frac{n}{5n+6}$

30. Terms greater than 5!

i.e., $(5!)^2, (6!)^2, \ldots, (100!)^2$ is divisible by 100

$\therefore$ For terms $(5!)^2, (6!)^2, \ldots, (100!)^2$

remainder is 0.

Now consider $(1!)^2 + (2!)^2 + (3!)^2 + (4!)^2$

$$= 1 + 4 + 36 + 576$$

$$= 617$$

When 617 is divided by 100, its remainder is 17.

$\therefore$ Required remainder is 17.

1. Given that

$G = \{1, 5, 7, 11\}$ is a group under multiplication modulo 12.

$\therefore$ $7^{-1} = 7$ $(\because 7^{-1} \otimes_{12} 7 = 1)$

Now, $7^{-1} \otimes_{12} (x \otimes_{12} 11) = 5$

$\Rightarrow$ $7 \otimes_{12} (11 \otimes_{12} x) = 5$

$\Rightarrow$ $(7 \otimes_{12} 11) \otimes_{12} x = 5$

$\Rightarrow$ $5 \otimes_{12} x = 5$

$\Rightarrow$ $x = 1$

2. In additive subgroup '0' is the identity element, but $0 \notin N$.

$\therefore$ Required set $(N, +)$ which is not subgroup.

3. We have, $\vec{\mathbf{p}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$, $\vec{\mathbf{q}} = 4\hat{\mathbf{k}} - \hat{\mathbf{j}}$ and $\vec{\mathbf{r}} = \hat{\mathbf{i}}\, 2\hat{\mathbf{r}}$

$\therefore$ $3\vec{\mathbf{p}} + \vec{\mathbf{q}} - 2\vec{\mathbf{r}}$

$= 3(\hat{\mathbf{i}} + \hat{\mathbf{j}}) + (4\hat{\mathbf{k}} - \hat{\mathbf{j}}) - 2(\hat{\mathbf{i}} + \hat{\mathbf{k}})$

$= \mathbf{i} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ Unit vector of $3\vec{\mathbf{p}} + \vec{\mathbf{q}} - 2\vec{\mathbf{r}}$

$$= \frac{\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{1^2 + 2^2 + 2^2}}$$

$$= \frac{1}{3}(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$$

4. Given that, $|\vec{\mathbf{a}}| = 3\sqrt{3}$, $|\vec{\mathbf{b}}| = 4$ and

$|\vec{\mathbf{a}} + \vec{\mathbf{b}}| = \sqrt{7}$

Now, $|\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2|\vec{\mathbf{a}}||\vec{\mathbf{b}}| \cos\theta$

$\Rightarrow (\sqrt{7})^2 = (3\sqrt{3})^2 + 4^2 + 2(3\sqrt{3})(4)\cos\theta$

$\Rightarrow$ $7 = 27 + 16 + 24\sqrt{3}\cos\theta$

$\Rightarrow$ $24\sqrt{3}\cos\theta = -36$

$\Rightarrow$ $\cos\theta = -\frac{3}{2\sqrt{3}} = -\frac{\sqrt{3}}{2}$

$\Rightarrow$ $\theta = 150^\circ$

35. Now, $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$

$= (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})\vec{\mathbf{c}} - (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{b}}$

$= \vec{\mathbf{0}} - \vec{\mathbf{0}}$ $(\because \vec{\mathbf{a}} \perp \vec{\mathbf{b}}$ and $\vec{\mathbf{a}} \perp \vec{\mathbf{c}}$

$= \vec{\mathbf{0}}$

36. We know, if $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are two adjacent sides of a parallelogram, then

Area $= |\vec{\mathbf{a}} \times \vec{\mathbf{b}}| = 15$ (given)...(i)

If the sides are $(3\vec{\mathbf{a}} + 2\vec{\mathbf{b}})$ and $(\vec{\mathbf{a}} + 3\vec{\mathbf{b}})$, then

Area of parallelogram

$= |(3\vec{\mathbf{a}} + 2\vec{\mathbf{b}}) \times (\vec{\mathbf{a}} + 3\vec{\mathbf{b}})|$

$= |7(\vec{\mathbf{a}} \times \vec{\mathbf{b}})|$

$= 7|\vec{\mathbf{a}} \times \vec{\mathbf{b}}|$

$= 7 \times 15$ (From (i))

$= 105$ sq unit

37. The locus of the point which moves such that the ratio of its distances from two fixed point in the plane is always a constant $k\,(k < 1)$ is an ellipse.

38. Since the lines $x + 3y - 9 = 0$, $4x + by - 2 = 0$ and $2x - y - 4 = 0$ are concurrent, then

$$\begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$$

$\Rightarrow 1(-4b - 2) - 3(-16 + 4) - 9(-4 - 2b) = 0$

$\Rightarrow$ $-4b - 2 + 36 + 36 + 18b = 0$

$\Rightarrow$ $14b + 70 = 0$

$\Rightarrow$ $b = -5$

39. We know, if the lines are perpendicular to each other, then $\theta = 90^\circ$

$\Rightarrow$ $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$

$\Rightarrow$ $\tan 90^\circ = \frac{2\sqrt{h^2 - ab}}{a + b}$

$\Rightarrow$ $\frac{1}{0} = \frac{2\sqrt{h^2 - ab}}{a + b}$

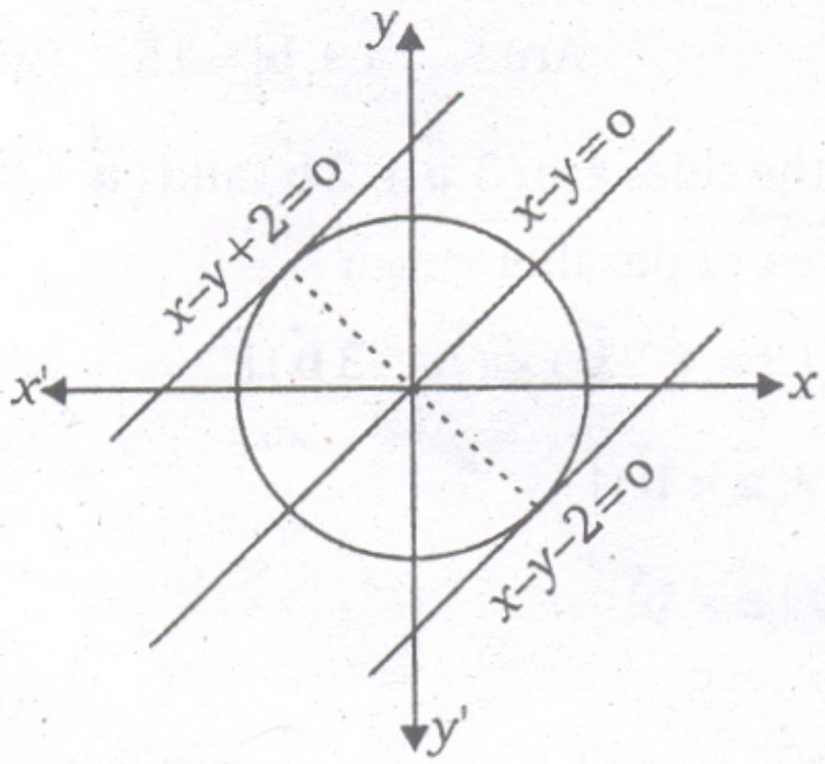
$\Rightarrow$ $a + b = 0$

40. Since the equations of tangents $x - y - 2 = 0$ and $x - y + 2 = 0$ are parallel.

$\therefore$ Distance between them = diameter of the circle

$$= \frac{2-(-2)}{\sqrt{1^2+1^2}} \qquad \left(\because \frac{C_2 - C_1}{\sqrt{a^2+b^2}}\right)$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$



$\therefore$ Radius $= \frac{1}{2}(2\sqrt{2}) = \sqrt{2}$

It is clear from the figure that centre lies on the origin.

$\therefore$ Equation of circle is

$$(x-0)^2 + (y-0)^2 = (\sqrt{2})^2$$

$$\Rightarrow \quad x^2 + y^2 = 2$$

41. Given equation of curve is

$$y = 2x^3 + ax^2 + bx + c \qquad \ldots(i)$$

Since it is passes through (0, 0)

$$\Rightarrow \quad 0 = 2(0) + a(0) + b(0) + c$$

$$\Rightarrow \quad c = 0 \qquad \ldots(ii)$$

On differentiating Eq. (i), we get

$$\frac{dy}{dx} = 6x^2 + 2ax + b$$

Since the tangents at $x = -1$ and $x = 2$ are parallel to x-axis.

$$\therefore \quad \frac{dy}{dx} = 0$$

At $x = -1$

$$6(-1)^2 + 2a(-1) + b = 0$$

$$\Rightarrow \quad 6 - 2a + b = 0 \qquad \ldots(iii)$$

At $x = 2$

$$6(2)^2 + 2a(2) + b = 0$$

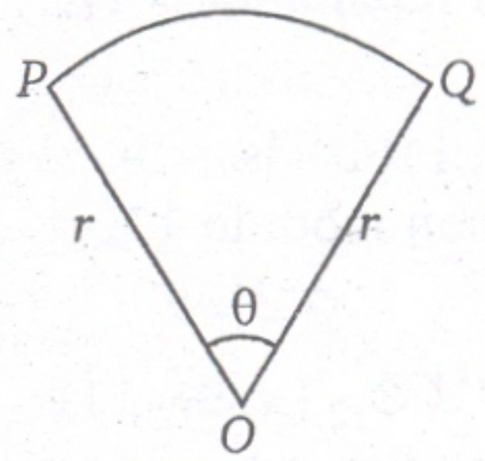
$$\Rightarrow \quad 24 + 4a + b = 0 \qquad \ldots(iv)$$

On solving Eqs. (iii) and (iv), we get

$$a = -3, \quad b = -12 \quad \text{and} \quad c = 0$$

42. Perimeter of sector $= 2r + r\theta$

$$\Rightarrow \quad 60 = 2r + r\theta \qquad \text{(given)}$$



$$\Rightarrow \quad \theta = \frac{60 - 2r}{r}$$

Area of sector, $A = \frac{\pi r^2 \theta}{360°}$

$$= \frac{\pi r^2 (60 - 2r)}{r\ 360}$$

$$= \frac{\pi r}{180}(30 - r)$$

$$\Rightarrow \quad \frac{dA}{dr} = \frac{\pi}{180}(30 - 2r)$$

For maximum area, $\frac{dA}{dr} = 0$

$$\Rightarrow \quad 30 - 2r = 0$$

$$\Rightarrow \quad r = 15$$

$$\therefore \quad \frac{d^2A}{dr^2} = \frac{\pi}{180}(0 - 2) = \frac{-\pi}{90} < 0$$

$\therefore$ It is maximum at $r = 15$ m.

43. Given equation of curve is $y = x^2 - x + 4$

Slope of tangent at $P(1, 4)$ is

$$\left(\frac{dy}{dx}\right) = 2x - 1$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(1,4)} = 2 - 1 = 1$$

$\therefore$ Equation of tangent is

$$y - 4 = 1(x - 1)$$

$$\Rightarrow \quad y - x = 3 \qquad \ldots(i$$

and equation of normal at point $P(1,4)$ is

$$y - 4 = -1(x - 1)$$

$$\Rightarrow \quad x + y = 5 \qquad \ldots(ii$$

Since the tangent cuts x-axis at A

$\therefore$ Co-ordinates of A are (–3, 0)

and the normal cuts x-axis at B

$\therefore$ Co-ordinates of B are $(5, 0)$

$\therefore$ Area of ΔPAB

$$= \frac{1}{2}\begin{Vmatrix} 1 & 4 & 1 \\ -3 & 0 & 1 \\ 5 & 0 & 1 \end{Vmatrix}$$

$$= \frac{1}{2}|[-4(-3-5)]| = \frac{1}{2}|32|$$

$$= 16 \text{ sq unit}$$

44. Let $I = \int \frac{(x^3 + 3x^2 + 3x + 1)}{(x+1)^5}\, dx$

$$= \int \frac{(x+1)^3}{(x+1)^5}\, dx = \int \frac{1}{(x+1)^2}\, dx$$

$$= -\frac{1}{(x+1)} + c$$

45. Let $I = \int \frac{\text{cosec } x}{\cos^2\left(1 + \log \tan \frac{x}{2}\right)} dx$

Put $\quad 1 + \log \tan \frac{x}{2} = t$

$$\Rightarrow \quad \frac{1}{\tan \frac{x}{2}} \cdot \sec^2 \frac{x}{2} \cdot \frac{1}{2}\, dx = dt$$

$$\Rightarrow \quad \frac{1}{2 \sin \frac{x}{2} \cos \frac{x}{2}}\, dx = dt$$

$$\Rightarrow \quad \text{cosec } x\, dx = dt$$

$$\therefore \quad I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t\, dt$$

$$= \tan t + c$$

$$= \tan\left(1 + \log \tan \frac{x}{2}\right) + c$$

46. Let $z = \frac{(-\sqrt{3} + 3i)(1-i)}{(3 + \sqrt{3}i)\, i\, (\sqrt{3} + \sqrt{3}i)}$

$$= \frac{(-\sqrt{3} + 3i)(1-i)}{(3i - \sqrt{3})\sqrt{3}\,(1+i)}$$

$$= \frac{1}{\sqrt{3}}\left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)$$

$$= \frac{1}{\sqrt{3}}\left(\frac{1-1-2i}{1+1}\right)$$

$$= -\frac{i}{\sqrt{3}}$$

The complex number z is represented on y-axis (imaginary axis).

47. Given that

$$2x = -1 + \sqrt{3}\, i$$

$$\Rightarrow \quad x = \frac{-1 + \sqrt{3}i}{2}$$

$$= \omega$$

$\therefore \quad (1 - x^2 + x)^6 - (1 - x + x^2)^6$

$= (1 - \omega^2 + \omega)^6 - (1 - \omega + \omega^2)^6$

$= (-2\omega^2)^6 - (-2\omega)^6 \qquad (\because 1 + \omega + \omega^2 = 0)$

$= 2^6\, \omega^{12} - 2^6 \omega^6$

$= 2^6(1) - 2^6(1) \qquad (\because \omega^3 = 1)$

$= 0$

48. Let $z = (1 + i\sqrt{3})^8$

$$= (-2)^8 \left(\frac{1 + i\sqrt{3}}{-2}\right)^8 = (-2)^8\, (\omega^2)^8$$

$$= 2^8\, \omega^{16} = 2^8 \omega \qquad (\because \omega^3 = 1)$$

$$= 2^8 \left(\frac{-1 + i\sqrt{3}}{2}\right)$$

$$= 2^8 \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$$

$\therefore$ Modulus $= 2^8 = 256$

and Amplitude $= \frac{2\pi}{3}$.

49. $\lim\limits_{x \to 0} \frac{5^x - 5^{-x}}{2x}$

Applying L- Hospital's rule

$$= \lim_{x \to 0} \frac{5^x \log 5 + 5^{-x} \log 5}{2}$$

$$= \frac{\log 5 + \log 5}{2} = \log 5$$

50. If a function $f(x)$ is continuous at $x = a$, then it may or may not be differentiable at $x = a$.

$\therefore$ Option (b) is correct.

51. Let $I = \int \frac{dx}{x\sqrt{x^6 - 16}}$

$$= \frac{1}{3} \int \frac{3x^2}{x^3 \sqrt{(x^3)^2 - 4^2}}\, dx$$

Put $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\therefore \quad I = \frac{1}{3}\int \frac{dt}{t\sqrt{t^2 - 4^2}}$

$= \frac{1}{3\times 4}\sec^{-1}\left(\frac{t}{4}\right) + c$

$= \frac{1}{12}\sec^{-1}\left(\frac{x^3}{4}\right) + c$

52. Since, $I_1 = \int_0^{\pi/2} x \sin x \, dx$

and $I_2 = \int_0^{\pi/2} x \cos x \, dx$

$\therefore \quad I_1 = \int_0^{\pi/2} x \sin x \, dx$

$\Rightarrow \quad I_1 = -[x \cos x]_0^{\pi/2} - \int_0^{\pi/2} -\cos x \, dx$

$= [0 - 0] + [\sin x]_0^{\pi/2}$

$= 1 \qquad \ldots(i)$

and $I_2 = \int_0^{\pi/2} x \cos x \, dx$

$= [x \sin x]_0^{\pi/2} - \int_0^{\pi/2} \cos x \, dx$

$= \left(\frac{\pi}{2} - 0\right) + [\sin x]_0^{\pi/2}$

$= \frac{\pi}{2} + 1 \qquad \ldots(i)$

From (i) and (ii)

$I_2 - I_1 = \frac{\pi}{2}$

53. Given that

$f(x) = 4x^2 - 3x + 1, g(x) = \frac{f(-x) - f(x)}{x^2 + 3}$

$\therefore \quad g(x) = \frac{(4x^2 + 3x + 1) - (4x^2 - 3x + 1)}{x^2 + 3}$

$= \frac{6x}{x^2 + 3}$

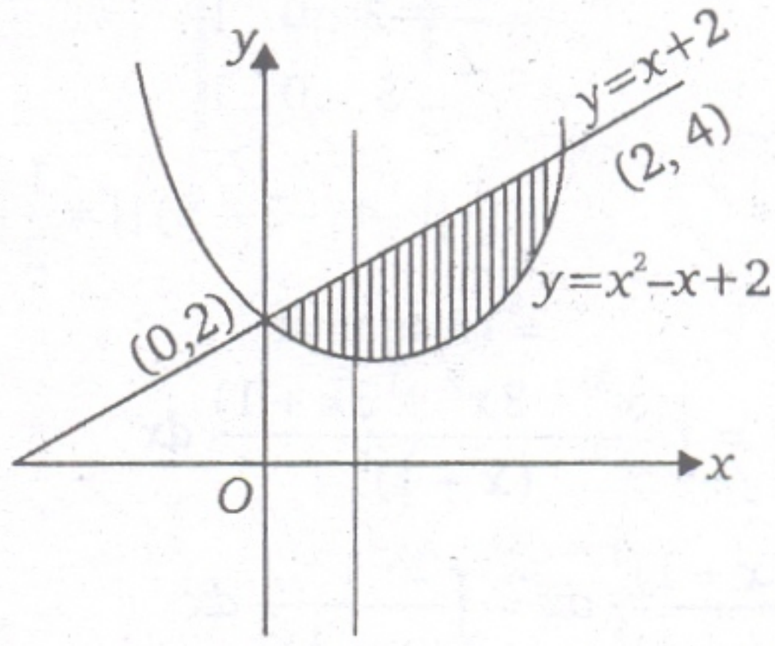
Now, $g(-x) = -\frac{6x}{x^2 + 3}$

$= -g(x)$

Which is an odd function

$\therefore \quad \int_{-2}^{2} g(x)\, dx = 0$

54. Given equation of parabola is



$y = x^2 - x + 2$

or $\left(x - \frac{1}{2}\right)^2 = y - \frac{7}{4}$

and equation of line is

$y = x + 2$

$\therefore$ Required area

$= \int_0^2 [(x + 2) - (x^2 - x + 2)]\, dx$

$= \int_0^2 (-x^2 + 2x)\, dx$

$= \left[-\frac{x^3}{3} + x^2\right]_0^2 = -\frac{8}{3} + 4$

$= \frac{4}{3}$ sq unit

55. We have

$e^{-x}\,(y + 1)\, dy + (\cos^2 x - \sin 2x) y \, dx = 0$

$\Rightarrow \frac{(y + 1)}{y} dy = -e^x\,(\cos^2 x - \sin 2x)\, dx$

$\Rightarrow \left(1 + \frac{1}{y}\right) dy = -e^x\,(\cos^2 x - \sin 2x)\, dx$

On integrating both sides, we get

$y + \log y = -e^x \cos^2 x + \int e^x \sin 2x \, dx$

$- \int e^x \sin 2x \, dx + c$

$\Rightarrow \quad y + \log y = -e^x \cos^2 x + c$